Working and Saving Informally: The Link between Labor Market Informality and Financial Exclusion.*

Preliminary and Incomplete

Luca Flabbi† Mauricio Tejada‡

November 15, 2022

Abstract

The high level of informality and the low level of savings observed in developing countries are fundamentally linked because informal workers have limited access to formal financial institutions. We study this link by developing and estimating a labor market model where workers can be employed both formally and informally and agents can save through both formal and informal financial institutions. We estimate the model on nationally representative data for Colombia and use the estimated model to simulate counterfactual experiments. Results show that reaching full financial inclusion of informal workers would increase savings by 3% a month and formal assets by 21%. The same policy would strongly decrease inequality in assets and mildly decrease inequality in consumption.

Keywords: Informality, financial inclusion, savings, labor market search, structural estimation. *JEL Codes:* J3, J64, O16, O17.

*We thank for their insightful comments the seminar participants at University of Cambridge and at Catholic University of Milan. Tejada gratefully acknowledges financial support from FONDECYT, grant project No. 11196296.

†Department of Economics, University of North Carolina - Chapel Hill, USA. E-mail: luca.flabbi@unc.edu.

‡Department of Economics, Universidad Alberto Hurtado, Santiago Chile. E-mail: matejada@uahurtado.cl.

1 Introduction

Developing countries are characterized by high informal employment and by low saving rates. While informality may introduce some useful flexibility in overly regulated labor markets, it also lowers workers' protections and increases their employment risks (World Bank, 2013; Perry et al., 2007; La Porta and Shleifer, 2014). A number of contributions has also shown that its presence distorts incentives and returns to productive investment decisions.[1] Low saving rates make individuals more vulnerable to shocks and the overall economy less resilient (Cavallo et al., 2016; Karlan and Morduch, 2010). The literature has also shown that low saving rates in low- and middle-income economies cannot simply be explained away by the presence of many individuals "too poor to save": limited access to financial services is playing a crucial role.[2]

If both high levels of informality and low levels of saving are problems in themselves, they are also strongly linked to each other. The presence of high informality increases the need for precautionary savings because of the higher employment risk faced by workers. But the informality status may also prevent workers to access the tools for effective saving. In turn, financial exclusion may induce workers to accept informal jobs with higher frequency because prevents them to save enough to finance an effective labor market search. This deep link introduces a vicious cycle that makes it extremely difficult to address a problem without considering the other. It may even prevent to know how important each of the channels is when considered in isolation. Despite this, the literature typically analyzes the high level of informality and the low level of savings as two separate issues.

This paper goes beyond this approach by developing an environment that integrates all the crucial elements giving rise to both phenomena. To this end, we develop a labor market model where workers can be employed both formally and informally and where agents can save through both formal and informal financial institutions. The reduced access to formal financial institutions captures the intensity of financial exclusion. To provide a quantitative assessment and to evaluate policy experiments, we estimate the model on data from Colombia. Colombia is a good candidate to conduct the analysis because it belongs to a region where both issues are particularly acute[3]

[1]The distortions affects both the firms side, with consequent loss of productivity (Ulyssea, 2018; de Paula and Scheinkman, 2010; La Porta and Shleifer, 2008), and the workers side, with consequent loss of human capital accumulation (Bobba et al., 2021, 2022).

[2]See Dizon et al. (2020); Dupas and Robinson (2013); Batini et al. (2010) for experimental evidence; Ogbuabor et al. (2013) for macro evidence; Bond et al. (2015) for model-based empirical evidence; Karlan and Morduch (2010) for a survey.

[3]Colombia is the fourth economy of the Latin America and the Caribbean (LAC) region. The informal sector employs between 30% and 80% of the workforce in the region (Gasparini and Tornarolli, 2009) and more than

and collects good quality data on both savings and labor market behavior.

Specifically, we develop an on-the-job search model of the labor market with savings. We extend the model developed by Lise (2013) by adding two types of jobs – formal and informal – and two types of assets – those accumulated using formal financial system and those accumulated using informal financial institutions (Eeckhout and Munshi, 2010). Financial exclusion is represented by the higher costs that workers employed informally face in accessing formal financial institutions. The equilibrium characterizes the distributions of individuals working as formal or informal employees, the portfolio choice between the two types assets, the saving rate, the unemployment rate and the total wealth level.

We estimate the model combining information from two household surveys for Colombia: The *Gran Encuesta Integrada de Hogares* (GEIH), a nationally representative monthly survey focusing on the labor market; and the *Encuesta Longitudinal Colombiana* (ELCA), a longitudinal survey focusing on consumption and saving behavior and access to financial services. The identification of the labor market parameters is relatively standard while the identification of the parameters related to financial exclusion is novel and relies on a parsimonious specification of portfolio allocation costs. The estimation results confirm the presence of a degree of financial exclusion for informal workers: they need to pay almost ten times as much as formal workers to maintain the average portfolio allocation of formal workers. They also show that formal workers receive better and more frequent job opportunities than informal workers.

We use the estimated model to perform counterfactual policy experiments. We find that providing full financial inclusion to informal workers would increase savings by 3% a month and formal assets by 21%. The same policy would also strongly decrease inequality in assets and mildly decrease inequality in consumption. We also evaluate a recent tax reform implemented in Colombia showing it had the potential to increase savings by 10% a month.

Our paper contributes to numerous strands of the literature. First, we contribute to the now large literature on savings in developing countries.[4] The literature has shown that the low level of savings is not simply driven by the population being "too poor to save" but it is significantly influenced by the institutional context. In it, the interplay of labor market informality and financial exclusion plays a prominent role but empirical results on this relation is mixed. In LAC, Lorenzo

40% of the world's GDP produced by the informal sector is produced in LAC (Schneider et al., 2010). Savings in the region equal 17% of GDP compared to the 30% of other middle-income countries (World Bank's World Development Indicators). The low saving rate has persisted despite policy efforts aimed at increasing it and despite relatively good macroeconomic conditions over a number of years (Reinhardt, 2008; Cavallo et al., 2016).

[4]See Karlan and Morduch (2010) for a general survey on saving and financial access; Karlan et al. (2014) for a survey focusing on saving by the poor; and Cavallo et al. (2016) for a flagship publication focusing on LAC.

and Osimani (2001) show that informal households in Uruguay have lower saving rates than their formal counterparts. Schclarek and Caggia (2015) use a financial survey for Chile to show that, controlling for other determinants of savings, informal households save, on average, less than formal households, more so in the case of precautionary savings. In contrast, Granda and Hamann (2020) use a financial survey for Colombia to find that the saving rate of those who are informally employed is higher than that of those who are formally employed. In other developing countries, Dupas and Robinson (2013) argue that informal households save less because they are financially excluded. The experiment they run in Kenya shows that providing a safe place to save increased health care savings by more than 60%, supporting the existence of capital market segmentation between the formal and the informal sectors (see also Batini et al. (2010)). From an aggregate perspective, Ogbuabor et al. (2013) use time series data for Nigeria showing that informality potentially hinders the growth of aggregate domestic savings due to the lack of a well-working mechanism of financial intermediation. Financial segmentation also implies that informal households have lower access to formal credit and depend strongly on informal sources of financing (Gatti and Honorati, 2008; Dabla-Norris and Koeda, 2008). Our contribution provides an explanation for these mixed empirical results since we are able to estimate a joint model of both informality and financial exclusion that endogenizes both labor market and saving decisions.

Second, our contribution relates to the growing literature using models with frictions to explain the equilibrium effects of labor market informality.[5] While this literature has been effective in producing counterfactual experiments and in assessing a wide range of issues – from the impact of enforcement to the effect on schooling decisions –, none of these contributions have been able to take into account saving behavior and therefore cannot assess the impact of the financial exclusion associated with informality.

Third, we contribute to the literature that analyzes savings behavior in the presence of idiosyncratic risk when labor markets are affected by frictions.[6] While previous contributions in this literature are the closest to our formal setting, they do not address labor and financial market informality nor focus on data from low- and middle-income countries.[7] But this focus is needed

[5]Important early contributions in the theoretical literature are Albrecht et al. (2009); Charlot et al. (2013); and in the macro literature is Bosch and Esteban-Pretel (2012). Contributions estimating this class of models on micro data are fewer and more recent, they include Meghir et al. (2015); Bobba et al. (2021, 2022).

[6]For papers estimating the structural parameters of the model, see Rendon (2006); Lentz (2009); Lise (2013); García-Pérez and Rendon (2020); Abrahams (2022); for seminal theoretical contributions, see Danforth (1979); Acemoglu and Shimer (1999); Browning et al. (2007); for macro-oriented contributions, see Krusell et al. (2010); Bils et al. (2011); Ji (2021); Setty and Yedid-Levi (2021); Pizzo (2022).

[7]One recent exception is Pierri and Kawamura (2022) studying the pension system in Chile. However, Chile is the only LAC country in which informality in the labor market is *not* a first order problem, as the authors

since workers in low- and middle-income countries face higher idiosyncratic risk, larger imperfection in capital markets and stronger frictions in the labor market than workers in high-income countries. The need is exacerbated by the fact that results from high-income countries do not readily transfer to the institutional context of poorer countries since informality and financial exclusion both increase the idiosyncratic risk and distort the saving rate necessary to deal with it. Indeed, when financial institutions function well in developing countries, they usefully complement social protection programs – which are usually at the center of the policy debate – in helping households to manage risk (Perry et al., 2007).

Finally, we contribute to the sparse literature that attempts to jointly analyze labor market informality and optimal saving decisions. Esteban-Pretel and Kitao (2021) develop and calibrate on macro data a model which includes labor market informality and savings but does not allow for financial exclusion.[8] Granda and Hamann (2020) also provide a macro contribution: they develop a somewhat more stylized labor market and allow for a link between labor market informality and some disadvantage in the financial market. However, they cannot introduce genuine financial exclusion because they assume only one type of asset.

The paper is organized as follows. In section 2 we describe the theoretical model, define the equilibrium, and briefly discuss the solution method. In section 4 we present the data, the identification, and the estimation method and results. In section 5 we analyze a series of counterfactual experiments, assessing the importance of labor market informality and financial exclusion on saving rates and inequality. Finally, 6 concludes.

2 The Model

The model needs to characterize the joint dynamic of labor market and saving decisions in presence of high levels of informality. Specifically, the labor market environment must generate an equilibrium where different types of contracts arise even if agents are ex-ante identical; the saving decisions setting should include a portfolio choice between at least two types of assets, one fully accessible and the other not.

A search model is a good candidate to describe a labor market with high informality: frictions allow different job contracts to survive in equilibrium and tractability delivers a dynamic model

acknowledge.

[8]Flórez (2017) is a theoretical paper that provides a similar earlier contribution to the one provided in Esteban-Pretel and Kitao (2021). As in their paper, the labor market includes an informal sector but the financial market does not.

that can be identified with data available in low- and middle-income countries.[9] Typical models in this area assume linear utility and ignore saving behavior. But a handful of contributions have been able to develop and estimate search models with saving and borrowing decisions.[10] However, they all focus on high-income countries and assume a very simple saving behavior: workers can only save in one asset. Therefore, we have to significantly extend the environment proposed by the previous literature in order to capture the crucial features we are interested in.

The first extension is to add the presence of two types of jobs – formal and informal – as done by previous contributions in the literature on search and informality. The second extension is to add the existence of two types of assets – those accumulated in the formal financial system and those accumulated outside of it (Eeckhout and Munshi, 2010) – a new extension that has never been combined with search models of the labor market.[11]

The resulting model's equilibrium characterizes the distributions of individuals working as formal or informal employees, the portfolio choice between the two types assets, the saving rate, the unemployment rate and the total wealth level. We assume as exogenous the wage offers distributions, some mobility parameters and the portfolio costs.

2.1 Environment

We assume continuous time and a stationary environment. Individuals discount the future at a common Poisson rate ρ and die at a common Poisson rate θ,[12] leading to the effective discount rate $\tilde{\rho} = \rho + \theta$. Individuals are ex-ante homogeneous but they differ ex-post in their labor market histories, and hence in the total wealth they are able to accumulate. Individuals are risk averse and derive utility from consumption. We also assume that markets are incomplete in the sense that individuals cannot fully insure against labor income risk: they can only (partially) self-insure by accumulating assets.[13]

[9]Previous contributions have exploited exactly these features to study Brazil (Meghir et al., 2015; Bosch and Esteban-Pretel, 2012), Mexico (Bobba et al., 2021, 2022), an average of large LAC countries (Albrecht et al., 2009).

[10]The completed and published contributions are: Rendon (2006); Lentz (2009); Lise (2013); García-Pérez and Rendon (2020). The closest to our setting is Lise (2013).

[11]A recent work in progress is attempting something similar in order to study labor market behavior and the holding of medical debt as a different negative asset from regular debt (Nishiyama, 2022).

[12]The presence of a death shock ensures stationarity in the model environment since we assume agents are born with zero assets.

[13]This is the standard assumption in economies with heterogeneous agents (Huggett, 1993; Aiyagari, 1994).

Individuals maximize the following additive separable expected lifetime utility function:

$$E_0 \int_0^\infty e^{-\tilde{\rho}t} \left[u(c) + \epsilon f\right] \tag{1}$$

where c is consumption, f is an indicator variable equal 1 if the individual is working formally, and $\epsilon > 0$ is the additional utility capturing all the benefits received by workers hired formally with respect to those hired informally. These are benefits that are not already captured by the labor market dynamic such as the higher and better social security benefits and the guarantees implied by labor regulations.[14] The instantaneous utility function satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. We follow the relevant literature[15] by assuming a Constant Relative Risk Aversion (CRRA) instantaneous utility function: $u(c) = \frac{c^\delta}{\delta}$.

The labor market is characterized by three states: unemployment, employment in a formal job, and employment in an informal job. Both unemployed and employed individuals search for jobs and receive offers. Unemployed workers receive a flow income b, which captures possible unemployment benefits and other transfers and subsidies. Jobs offers arrive at Poisson rate λ^u to the unemployed and at Poisson rate $\lambda^e(f)$ to the employed. We denote the measures of workers in unemployment with v and in employment with $e(f)$. A job offer is a pair $\{w, f\}$ denoting the wage being offered w for a job with formality status f. Wages are draws from the exogenous mixture distribution $\sum_{f=0,1} F(w|f)p(f)$. Jobs are terminated at the exogenous Poisson rate $\eta(f)$.

The financial market is characterized by two types of liquid assets. The first asset, denoted with a_1, is a standard risk-less asset with rate of return r_1. This is the asset provided by formal financial institutions and, as specified below, its accessibility depends on the labor market state. The second asset, denoted by a_2, is a risky asset and has rate of return denoted by r_2. This is the asset provided by informal financial institutions and it is equally accessible to anyone. We model the rate of return of the risky asset by assuming a Ornstein-Uhlenbeck process, or elastic random walk process, (Vasicek, 1977; Munk and Sørensen, 2010) such that:

$$dr_2 = \kappa(\bar{r}_2 - r_2)dt + \sigma_z dz \tag{2}$$

where z is a standard Brownian motion with $dz = \varepsilon_t\sqrt{dt}$ and $\varepsilon_t \sim \mathcal{N}(0, 1)$; and κ, $\bar{r}_2$, and σ_z are positive constants. This process is the analogue of an AR(1) stationary process with autocorrelation $e^{-\kappa} \approx 1 - \kappa$. The instantaneous drift $\kappa(\bar{r}_2 - r_2)$ represents the force that keeps

[14] In this parameterization we follow Conti et al. (2018) in their study of labor market informality in Mexico. The assumption of separability was already present in Dey and Flinn (2008a) in their contribution on employer-provided health insurance and labor market search.

[15] See for example Dey and Flinn (2005); Krusell et al. (2010); Lise (2013).

the process around the long term mean $\bar{r}_2$, while the stochastic component with variance s^2 generates the erratic but continuous fluctuations of the process from the mean. In steady state, the distribution of the rate of return of the risky informal asset is $\mathcal{N}\left(\bar{r}_2, \frac{\sigma_z^2}{2\kappa}\right)$. This variation of returns represents the main feature of informal financial institutions. Since they are not registered by governments, they lack proper supervision and regulation, leading to higher proportions of risky behavior by operators and higher incidence of financial misdemeanor. Still, reputation and other market mechanisms avoid huge swings in returns, keeping the process stationary and guaranteeing a certain degree of persistence (Cavallo et al., 2016; Adentiloye, 2006; World Bank, 1989).

We denote total wealth for a given individual as $a \equiv a_1 + a_2$ and total wealth distribution in the aggregate with $\Lambda(a)$. The share of the formal asset in the total wealth is defined as $\phi \equiv \frac{a_1}{a}$. As in Bonaparte and Cooper (2009) and Bonaparte et al. (2012), we assume that individuals are able to decide their portfolio composition but also that maintaining a given composition has a cost. Specifically, we assume a convex cost function and we allow it to depend on the agent's labor market status. The functional form is equal to $\frac{\psi^u}{2}\phi^2$ for the unemployed and to $\frac{\psi^e(f)}{2}\phi^2$ for the employed.[16] The differential access to the formal financial system is therefore captured by the ψ parameters. For example, the partial exclusion experienced by informal workers is captured by $\psi^e(0) > \psi^e(1)$. This parameterization implies that informal workers are not fully excluded from the formal financial system but that they incur a higher cost in maintaining a positive proportion of their assets there, an interpretation fully consistent with the empirical evidence in the LAC region.[17] Financial institutions charge a markup $(1+\nu)$ over the savings rate of return when they lend. Therefore, the interest rate spreads are νr_i, $i = 1, 2$ for all workers in any labor market state.

When an agent dies, her assets are *not* passed on to the next generation: the newborn agent starts life as unemployed with zero assets. Following the Merton (1971) derivation for a multi-asset optimal decision problem, the individual budget constraint can then be written as:

$$da = \begin{cases} \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + b - c - \frac{\psi^u}{2}\phi^2\right] dt & \text{if unemployed} \\ \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] dt & \text{if employed} \end{cases} \tag{3}$$

where I_{a^-} is an indicator variable that takes the value of 1 if $a < 0$ (debt) and 0 otherwise, and τ is the pay-roll contribution paid by workers who are employed formally. As in Aiyagari (1994)

[16] Instead of assuming non-convex portfolio costs as in Bonaparte et al. (2012), we simplify the analysis by assuming a convex cost function. Our setting is convenient because it generates a well-defined decision rule for the portfolio choice while keeping the model tractable without losing the Bonaparte et al. (2012)'s result of the portfolio cost being the source of partial segregation in the financial market.

[17] For evidence on LAC see Cavallo et al. (2016). But the evidence is also found in low- and middle-income countries in other world regions (see for example Eeckhout and Munshi, 2010; Adentiloye, 2006).

and Lise (2013), we assume a borrowing constraint $a \geq \underline{a}$, where $\underline{a}$ is a self-imposed borrowing limit for a permanent state of unemployment. We assume it to be equal to the self-imposed limit in the worse case scenario, i.e. the scenario where, on top of being permanently unemployed, the individual has all the (negative) assets in the informal financial system and faces a 'very high' interest rate that we denote with $\bar{\bar{r}}_2$.[18] This scenario implies:

$$\underline{a} = -\frac{b}{\bar{\bar{r}}_2(1+\nu)} \tag{4}$$

2.2 Value Functions

Given an initial distribution of the stock of assets, the individual's problem is to decide if accepting or rejecting job offer and to choose the paths of consumption c and portfolio composition ϕ, in order to maximize (1) subject to (3) and (4). This problem can be conveniently represented by a set of stochastic Hamilton-Jacobi-Bellman equations for each labor market state.[19] We denote with $U(a, r_2)$ the value of being unemployed with a stock of assets of a and facing a rate of return r_2 for the informal asset. We denote with $W(a, r_2, w, f)$ the value of being employed in job type f, receiving a wage w with a stock of assets of a and facing a rate of return r_2.

The steady state value of unemployment $U(a, r_2)$ satisfies:

$$\begin{aligned} \tilde{\rho} U(a, r_2) \;\; = \;\; & \max_{0 \leq c \leq \bar{c}, 0 \leq \phi \leq 1} \Bigg\{ u(c) + \partial_a U(a, r_2) \left[(r_1 \phi + r_2(1-\phi))(1 + \nu I_{a^-}) a + b - c - \frac{\psi^u}{2} \phi^2 \right] \\ & + \partial_{r_2} U(a, r_2) \kappa(\bar{r}_2 - r_2) + \frac{1}{2} \partial^2_{r_2} U(a, r_2) \sigma_z^2 \\ & + \lambda^u \sum_{f=0}^{1} \left(\int_w \max\{W(a, r_2, w, f) - U(a, r_2), 0\} dF(w|f) p(f) \right) \Bigg\} \end{aligned} \tag{5}$$

Equation (5) indicates that unemployed individuals receive a flow utility $u(c)$ plus the expected change in the value of non-employment. The expected change is comprised of three parts.[20] First, the value changes because individuals accumulate (or decumulate) assets by the amount $\frac{da}{dt}$, which is valued at the marginal value of assets $\partial_a U(a, r_2)$. Second, the value changes because deviations of the rate of return of the informal asset from its long term mean are corrected by

[18] We define $\bar{\bar{r}}_2$ as the upper bound of the confidence interval that contains 99% of the r_2 draws. This assumption has the advantage of delivering tractability while maintaining a reasonable interpretation. With our two assets setting, exactly following Lise (2013) would have led to $\underline{a} = -\frac{b - \frac{\psi^u}{2}\phi^2}{(r_1\phi + r_2(1-\phi))(1+\nu)}$. But this expression depends on the endogenous variable ϕ, which implicitly depends on the state variable a. This dependence has the much less tractable implication of making the boundary of the problem individual specific.

[19] For a detailed derivation of the Hamilton-Jacobi-Bellman equations see Appendix A.1.

[20] We define $\partial_a U(a, r_2) \equiv \frac{\partial U(a, r_2)}{\partial a}$, $\partial_{r_2} U(a, r_2) \equiv \frac{\partial U(a, r_2)}{\partial r_2}$, and $\partial^2_{r_2} U(a, r_2) \equiv \frac{\partial^2 U(a, r_2)}{\partial r_2^2}$.

the amount $\frac{dr_2}{dt}$, which is valued at the marginal value of the rate of return $\partial_{r_2} U(a, r_2)$. The deviation also generates disutility due to the uncertainty generated by shocks on the rate of return of the informal asset: the disutility is captured by the diffusion term $\partial^2_{r_2} U(a, r_2)\sigma_z^2$. Third, the value changes because individuals receive job offers and may move from non-employment to employment as a result. Job offers (w, f) arrive at the rate λ^u and, if acceptable, generate a value gain equal to $W(a, r_2, w, f) - U(a, r_2)$.

Similarly, the steady state value of employment $W(a, r_2, w, f)$ satisfies:

$$\begin{aligned}\tilde{\rho} W(a, r_2, w, f) \;=\; & \max_{0 \leq c \leq \bar{c}, 0 \leq \phi \leq 1} \Big\{ u(c) + \epsilon f + \partial_a W(a, r_2, w, f) \left[(r_1 \phi + r_2(1-\phi))(1 + \nu I_{a^-}) a \right. \\ & \left. + w(1 - \tau f) - c - \frac{\psi^e(f)}{2} \phi^2 \right] + \partial_{r_2} W(a, r_2, w, f) \kappa(\bar{r}_2 - r_2) \\ & + \frac{1}{2} \partial^2_{r_2} W(a, r_2, w, f) \sigma_z^2 + \eta(f) \left[U(a, r_2) - W(a, r_2, w, f) \right] \\ & + \lambda^e(f) \sum_{f=0}^{1} \left(\int_{w'} \max\{ W(a, r_2, w', f') - W(a, r_2, w, f), 0 \} dF(w'|f') p(f') \right) \Big\}\end{aligned} \tag{6}$$

Equation (6) indicates that employed individuals receive a flow utility $u(c)$ plus the expected change in the value of employment, which is now comprised of four components.[21] The first two components – $\partial_a W(a, r_2, w, f) \frac{da}{dt}$ and $\partial_{r_2} W(a, r_2, w, f) \frac{dr_2}{dt}$ plus the diffusion term – have the same interpretation as in equation (5). The third one generates a a value loss equal to $U(a, r_2) - W(a, r_2, w, f)$ and occurs when a termination shock hits, an event arriving at the rate $\eta(f)$. The fourth component can potentially generate a value gain of $W(a, r_2, w', f') - W(a, r_2, w, f)$, an event occurring when an employed individual earning w in a job type f receives a better job offer (w', f'). Job offers are received on the job at the rate $\lambda^e(f)$.

2.3 Decision rules

The optimal consumption and portfolio decision rules are derived from the first order conditions of equations (5) and (6), that is:

$$u'(c) = \partial_a U(a, r_2) \tag{7}$$

$$(r_1 - r_2)(1 + \nu I_{a^-}) a = \psi^u \phi \tag{8}$$

$$u'(c) = \partial_a W(a, r_2, w, f) \tag{9}$$

$$(r_1 - r_2)(1 + \nu I_{a^-}) a = \psi^e(f) \phi \tag{10}$$

[21] We define $\partial_a W(a, r_2, w, f) \equiv \frac{\partial W(a, r_2, w, f)}{\partial a}$, $\partial_{r_2} W(a, r_2, w, f) \equiv \frac{\partial W(a, r_2, w, f)}{\partial r_2}$, and $\partial^2_{r_2} W(a, r_2, w, f) \equiv \frac{\partial^2 W(a, r_2, w, f)}{\partial r_2^2}$.

Equations (7) and (9) are the standard inter-temporal conditions for unemployed and employed individuals, indicating that the marginal utility of consumption is equal to the marginal value of assets. These conditions imply that the optimal rules for consumption are $c^u(a, r_2) = u'^{-1}(\partial_a U(a, r_2))$ and $c^e(a, r_2, w, f) = u'^{-1}(\partial_a W(a, r_2, w, f))$. Equations (8) and (10) establish that the optimal portfolio allocation is obtained by equating the marginal benefit of adjusting the portfolio to its marginal cost. Notice that the marginal benefit depends on the differential return. The resulting optimal rules for the portfolio decisions are $\phi^u(a, r_2) = \frac{(r_1 - r_2)(1+\nu I_{a^-})a}{\psi^u}$ and $\phi^e(a, r_2, f) = \frac{(r_1 - r_2)(1+\nu I_{a^-})a}{\psi^e(f)}$. Corner solutions on 0 and 1 are possible on these rules; the solution is interior whenever $0 < (r_1 - r_2)(1 + \nu I_{a^-})a < \psi$.

The optimal labor market decision rules about accepting or rejecting job offers have the usual form. An unemployed individual with assets a and facing a rate of return r_2 for informal assets accepts a job offer $\{w, f\}$ if $W(a, r_2, w, f) \geq U(a, r_2)$ and rejects otherwise. In turn, an employed individual with assets a and facing a rate of return r_2 for informal assets accepts a job offer $\{w', f'\}$ if $W(a, r_2, w', f') \geq W(a, r_2, w, f)$.

2.4 Equilibrium and solution method

The steady state equilibrium in the model is defined as:

Definition. Given the primitive parameters $\{\rho, \theta, \lambda^u, \lambda^e(1), \lambda^e(0), \eta(1), \eta(0), \psi^u, \psi^e(1), \psi^e(0), b\}$, the instantaneous utility function $u(c)$, the distributions of wage offers $F(w|1)$, $F(w|0)$, $p(1)$,[22] the *steady state equilibrium* is a set of value functions $U(a, r_2)$ and $W(a, r_2, w, f)$ that satisfy the Hamilton-Jacobi-Bellman equations (5) and (6), together with the invariant distributions of individuals across labor market states $\{u, e(1), e(0)\}$ and of total assets $\Lambda(a)$.

We use a two-step approach to solve the model. In the first step, we solve the Hamilton-Jacobi-Bellman equations using a value function iteration method with an upwind scheme, while in the second step we solve for the invariant distributions of labor market states and of total assets by simulation. The full derivation is in Appendix A.2.

3 Data

We use data from a relatively large LAC country where working in the informal sector and saving outside formal financial institutions is common and widespread: Colombia. While other medium

[22] Where $p(0) = 1 - p(1)$.

and large LAC countries share these features, Colombia has the advantage of providing researchers with good quality data on both labor market and saving behavior.[23]

3.1 Data Sources

We combine information from two data sources: a standard labor market survey – the *Gran Encuesta Integrada de Hogares* (GEIH) – and a survey focusing on individuals' saving and borrowing behavior – the *Encuesta Longitudinal Colombiana* (ELCA). We focus on 2016 since it is the last year for which both surveys are available together. We need to combine both datasets because the first, while giving a good description of the labor market, does not contain information on saving and borrowing; and the second, while collecting individuals' saving and borrowing information, does not contain enough information on the labor market dynamic.

The GEIH is a nationally representative survey collected monthly by the *Administrative Department of National Statistics* (DANE). The survey contains individual characteristics, such as gender, age, and schooling; and provides labor market outcomes, such as employment status, durations, monthly labor income, weekly hours worked, and occupational characteristics. We pool together all the surveys from January to December of 2016. To be consistent with the theoretical model, we extract an estimation sample relatively homogeneous over the main demographic characteristics. We focus on individuals who are: male, between 25 and 55 years old, living in urban areas, with only secondary education completed ("unskilled"), and working full-time when employed.[24] These controls generate a sample of individuals for whom informality both in the labor and in the financial market is likely to be a relevant issue. It is also the demographic group that constitutes the main component of the Colombian labor force.

We define employed workers to be *informal* when they do not contribute to social security, a description consistent with the International Labor Organization (ILO)'s definition and with the definition used by the literature on the region (Perry et al., 2007; Kanbur, 2009; Bobba et al., 2022). These workers are composed of two groups. Workers that, while informal, are in a subordinate working relationship with a well-defined employer; and workers that, while still depending on others for their employment, are occupied in an activity with more independence, such as selling cheap goods in a street corner. This second group is sometime considered a separate labor market state defined as "necessity" self-employment (Bobba et al., 2022; Narita, 2020). Following the majority of the literature, we do *not* introduce this distinction and consider

[23]Chile has a similar data availability but experiences much milder labor market informality.

[24]Full-time is defined as working 40 or more hours per week (top-coded at 100 hours per week).

both groups as simply informal workers.[25] We define individuals as *unemployed* if they are classified as not working and actively searching for a job in the labor market. The labor income variable we use is the gross real monthly wage expressed in December 2016 US dollars. GEIH is a cross-sectional survey so we cannot rely on a panel structure, not even on a rotating panel structure as the one in the US *Current Population Survey*, to collect information on individuals' labor market dynamic. Fortunately, the survey contains unemployment and employment on-going durations information.

The ELCA is a longitudinal survey that follows a representative sample of about 10,000 households in rural and urban areas every three years; the available waves are 2010, 2013, and 2016. This survey is carried out by the *Center for Studies on Economic Development* (CEDE) of the School of Economics at the *Universidad de los Andes*. It is part of a project designed to follow individuals for 12 years in order to collect information on a wide range of issues including income, consumption, and access to financial services.[26] Crucially, the survey contains the information missing from GEIH: individuals' saving and borrowing decisions. Individuals report whether they are able to save. If they do, they report average monthly savings and the proportion of their savings that are in formal or informal financial institutions. We define the first as banks, employee funds, credit unions and similar; we define the second as cash, informal group savings (such as ROSCA funds),[27] and similar.

3.2 Descriptive Statistics

Table 1 presents descriptive statistics on labor market outcomes computed using the estimation sample extracted from the GEIH survey. It shows that informality is a very important phenomenon in the sample of unskilled men since more than 50% of them work informally. A little less than 40% are hired formally and 7.7% are unemployed. Workers earn more, on average, if they are hired formally: average monthly wages are US$328 for formal workers and US$243 for informal workers, being the wage gap 35%. But the two wage distributions substantially overlap: about 30% of informal workers earn more than the median wage for formal workers. With respect to

[25] The typical definition of informal workers in the region sums up self-employed workers who are not professionals or technicians with salaried workers with jobs not registered with the social security system. This is basically the definition that we adopt. For papers using the same definition, see for example Conti et al. (2018); Meghir et al. (2015); Bosch and Esteban-Pretel (2012).

[26] Detailed information on the survey is available at: https://encuestalongitudinal.uniandes.edu.co/en/.

[27] ROSCA stands for *ROtating Savings and Credit Association*, i.e. a group of individuals acting as an informal financial institution by setting up a common fund via set contributions and withdrawals

durations, in our sample unemployed workers search for a job for 4 months on average, while workers maintain their current jobs for 68 and 90 months on average if they work formally and informally, respectively. Notice that informal jobs last 30% more than their formal counter parts. In addition, the distributions of durations is more spread out for the case of informal workers.

Descriptive statistics on savings behavior are presented in Table 2 and confirm well-known facts in the literature. Formal workers save a little bit more than informal workers: 27% of them have positive saving and save on average US$60 per month (leading to a saving rate of 13.3%). All these values are lower for informal workers. But where the difference is most striking is in the access to formal financial institutions: Almost 50% of formal workers put most of their saving in formal financial institutions while only 18% of informal workers do. Unsurprisingly, the unemployed save significantly less than both formal and informal workers but, when they save, they put their savings in formal financial institutions in higher proportions than informal workers (33%). This is a crucial feature that our model is able to account for thanks to the forward looking behavior and the search frictions that we assume. For example, individuals currently unemployed may had spells in formal employment in the past and therefore had relatively convenient access to formal financial institutions.

4 Estimation

4.1 Identification discussion

We provide a heuristic identification discussion to clarify which data features identify the parameters of interest and which parameters cannot be identified with the data at our disposal and therefore need to be calibrated. A formal discussion of the identification of all the structural parameters is not feasible due to the highly non-linear nature of the model and of its mapping to sample moments.

First, we add one crucial functional form assumptions. Flinn and Heckman (1982) show that without additional assumptions on the wage offers distributions this class of models is not identified. We follow the literature[28] by assuming that both the formal and informal wage offers distribution are lognormal: $\log(w)|f \sim \mathcal{N}(\mu(f), \sigma(f))$, with $f = 0, 1$. Under this assumption, the primitive wage offers distributions can be identified by observing their truncations. Since, under the model, the observed wages distributions for formal and informal workers are the wage

[28] See the review article Eckstein and van den Berg (2007) but also some more recent works such as Bobba et al. (2022); Tejada et al. (2021); Flinn and Mullins (2015). Even if in these more recent contributions lognormality is assumed on another source of heterogeneity that then generates lognormal wage distributions.

offers truncated at the reservation wages, we can identify $(\mu(f), \sigma(f))$ directly from them. Since we observe the proportion of accepted jobs that are formal or informal, we can also recover $p(f)$. In addition, b and ϵ affect the reservation wage and therefore the the shape of the accepted wages distribution at at the bottom; hence to identify these parameters we use as relevant information in the data the bottom 5% of the observed wages distributions by type of job (note that b is shared by formal and informal workers, while ϵ only affects to formal workers).

Second, Flinn and Heckman (1982) also show that labor market dynamic information – as represented by durations – identifies the mobility parameters. In our sample, on-going durations in both employment and unemployment are available and, thanks to stationarity, they contribute the equivalent amount of information as complete spells of employment and unemployment. Unemployment durations contribute to the identification of λ^u while employment durations to the joint identification of $\{\eta(f), \lambda^e(f)\}$. To attain the separate identification of these last two sets of parameters, we exploit the steady state proportions in each labor market state. A more formal and detailed discussion is reported in Appendix B.

Third, we focus on the parameters of the portfolio cost functions: ψ^u and $\psi^e(f)$ (see Section 2.1). Given an initial distribution of assets, the optimal consumption and portfolio decisions determine the flow of savings da/dt, which in turn generates the steady states distribution of assets after converging over time. Therefore, the relevant pieces of information to identify the steady state equilibrium assets distribution, given the structure of the model, are the initial distribution of assets and the change in assets in equilibrium. In the data, we observe who saves, and if they do, how much they save. As a result, we are able to exploit the information contained in the distribution of savings by labor market states (in particular, we use the mean and the standard deviation of that distribution). The piece of information that is missing is the initial distribution of assets, hence we assume that all workers start with zero assets.[29] In turn, the optimal portfolio decision in equilibrium depends on the differential return, the cost of portfolio cost parameter, and the steady state distribution of assets. In the data, we observe whether individuals keep the majority of their assets in formal financial institutions according to their labor markets states, which are informative of the portfolio costs parameter in that state given the steady state distribution of assets and the differential returns.

Fourth, we discuss the identification of the rate of returns in both formal and informal institutions. For the formal asset rate of return, we use the yearly rate of return of a 10-year Colombian Government Bond. It was 7.5% in 2016, implying $r_1 = 0.075$. The informal asset rate of return is

[29]We also tried an alternative assumption on the initial distribution of assets, based on the observed distribution of bank assets available in the Colombian *Encuesta de Carga Financiera y Educación Financiera de los Hogares* of 2016. The results of the estimation were very similar.

more challenging since there is no credible and systematic information on how informal financial institutions behave. To identify the parameters of the r_2 process $\{\kappa, \bar{r}_2, \sigma_z\}$, we follow Eeckhout and Munshi (2010). They find that the implied rate of return for chit funds in South India, given a stable relationships in a matching equilibrium with informal financial institutions, is around 2.1 times the rate of return of formal financial institutions. If we apply the same proportion to our data, the maximum implied rate of return that an informal asset could pay becomes 15.8%. We use this information to assume that that 99% of the time r_2 belongs to the interval $[0, 0.158]$. Using the fact that r_2 is distributed as $\mathcal{N}\left(\bar{r}_2, \frac{\sigma_z^2}{2\kappa}\right)$, we recover $\bar{r}_2 = 0.079$ and $\frac{\sigma_z}{\sqrt{2\kappa}} = 0.031$. The only parameter left to be identified to fully describe the process for r_2 is therefore κ (see equation 2). It is a parameter shared by workers in all labor market states. As a result, information on the savings and portfolio allocations observed in our data, together with the restrictions implied by the model, is enough to identify it.

Fifth, we discuss the identification of the discount rate, the death rate and the relative risk aversion parameter. As shown in Section 2.1, the discount rate ρ and the death rate θ combine to create the effective discount rate $\tilde{\rho} = \rho + \theta$. As shown by Flinn and Heckman (1982), $\tilde{\rho}$ can only be jointly identified with the flow value of unemployment $u(b)$. Since we do not have a reliable source to compute b, we follow the literature[30] by calibrating the effective discount rate $\tilde{\rho}$ in order to identify b. We calibrate the discount rate ρ at the discount rate recommended for Latin America by multilateral development banks: 12% a year (Moore et al., 2020). We calibrate the death rate θ at 0.013 based on Colombia's life expectancy of 77 years.[31] The relative risk aversion parameter is notoriously difficult to identify in this class of models without additional specific sources of information. Flabbi and Mabli (2018) accomplish its identification by adding moments from spouses' household interaction; Dey and Flinn (2008b) by using information on employer-provided health insurance. Lacking additional information and lacking credible estimates of the parameter specific for Colombia, we follow Bond et al. (2015) – a paper estimating structural parameters of the Colombian economy – and fix the risk aversion parameter at 1.5, implying a calibration of our parameter δ equal to -0.5. This calibration is also very similar to the value estimated in Lise (2013), which delivers a risk aversion parameter equal to 1.455, and it is among the "consensus" values used in the literature.

Finally, we calibrate the two institutional parameters: τ, the pay-roll contribution paid by formal workers; and ν the markup that financial institutions charge when they lend to workers with respect to the rate they pay on worker's saving. Following Fernández and Villar (2017), we set $\tau = 0.16$; based on the IMF *International Financial Statistics*, we set $\nu = 1.14$.

[30] See for example Flinn and Heckman (1982); Eckstein and van den Berg (2007); Flinn (2002).

[31] Value reported by the World Development Indicators (WDI) compiled by the World Bank.

To summarize, based on the model presented in Section 2 and on the additional functional forms assumptions presented in this Section, the complete set of parameters we can identify with the data at our disposal is:

$$\Xi \equiv \{b, \epsilon, \lambda^u, \psi^u, \kappa, p(1)\} \cup \{\lambda^e(f), \eta(f), \mu(f), \sigma(f), \psi^e(f)\}_{f\in\{0,1\}}$$

In addition, we calibrate the parameters $\{r_1, \bar{r}_2, \sigma_z, \rho, \theta\}$ and we fix at their institutional value the parameters $\{\tau, \nu\}$.

4.2 Estimator

We estimate the parameters of the model using the Method of Simulated Moments (MSM). This method is commonly used to estimate highly non-linear models like ours (Gouriéroux and Monfort, 2002). Given Ξ the set of parameters to be estimated, M_N^D denotes the set of appropriately chosen statistics derived from our data sample of size N, and $M_T(\Xi)$ denotes the corresponding set of simulated statistics extracted from a sample of size T obtained from the steady state equilibrium implied by Ξ. Then our MSM estimator $\hat{\Xi}$ satisfies:

$$\hat{\Xi}_{N,T}(W) = \text{argmin}_{\Xi} \frac{1}{2} \left[M_N^D - M_T(\Xi)\right]' W_N \left[M_N^D - M_T(\Xi)\right] \quad (11)$$

where W is a symmetric, positive-definite weighting matrix.[32]

The set of chosen moment statistics in equation (11) are: the proportion of individuals in each labor market states; the average, the standard deviation, and the 5% percentile of the observed wages distributions for formal and informal jobs; the average and standard deviation of the on-going durations in unemployment, in formal employment, and in informal employment; the proportion of individuals in each labor market state who have more than 50% of their assets in the form of formal assets and the same proportions by quartiles of the formal and informal accepted wage distributions; and finally, the average and the standard deviation of the distributions of savings by labor market state and the average by quartiles of the formal and informal accepted wage distributions. The procedure generates a total of 40 moments that we use to estimate 16 parameters.

We use the Nelder-Mead simplex algorithm to minimize equation (11) and bootstrap to compute standard errors

[32]The inverse of the bootstrapped variance of each moment in the sample is typically used to construct the weighting matrix. Since our moments are computed using two sources of data with considerably different sample sizes, in the estimation we use the identity matrix instead.

4.3 Results

Table 3 reports the estimated parameters. In the top panel we collect the mobility parameters. The point estimates of the arrival rates of job offers imply that workers receive an offer on average every 5.6 months when they are unemployed and significantly less often when they are employed: respectively, every 29 months when formal and every 67 months when informal. The point estimates of the termination shocks imply that jobs are exogenously terminated every 5 years on average when working formally and every 6 years on average when working informally.

In the second panel we collect the wage offers parameters. The point estimate of $p(1)$ indicates that 46% of the wage offers are for formal jobs. The estimates of the location and scale parameters of the wage offers distributions imply an average offered wage of $311 and $242 per month for, respectively, formal and informal jobs (see top panel of table 4). The standard deviation of wage offers is also higher for formal jobs: 127 compared to 103 for informal jobs. These results imply that the wage offers distribution of formal jobs stochastically dominates the one of informal jobs, an implication that is consistent with the literature on informality in LAC.[33]

The third panel presents the estimated parameters of the portfolio cost function. They capture the extent of the exclusion of informal workers from formal financial institutions. The point estimates imply a cost parameter for informal workers that is nine times higher than the one for formal workers. Figure 1 gives a concrete representation of what this value means by showing the portfolio cost function for the whole range of the share of formal assets $\phi \equiv \frac{a_1}{a}$. Consider a worker holding the average portfolio allocation of formal workers: it is a value equal to 43% and corresponds in the figure to the dotted red vertical line. If such a worker holds a formal job, he will pay $0.22 per month in portfolio costs. But if he holds an informal job, he will pay $2.1 per month, a value almost ten times larger. This is one of the reasons why informal workers hold on average a lower proportion of their savings in the formal asset: the average ϕ for informal workers is about half the one for formal workers. But even at that much lower value, they pay an average fee which is higher than the one paid by formal workers (compare the intersection of the blue dotted and continuous line with the intersection of the red dotted and continuous line.)

The fourth panel presents the estimated parameters for the process of the rate of return of the informal asset. The estimated values imply that in steady state (as shown at the bottom of Table 4) the informal asset return has a mean of 7.9% a year and a standard deviation of

[33]An implication of both Meghir et al. (2015) and Bobba et al. (2022) is that the average lower level of productivity of informal jobs will be reflected in lower average job offers for informal workers. Tejada et al. (2021) provide direct estimates of wage offers distributions for formal and informal jobs in four LAC countries and confirm the ranking found here for Colombia.

3.1 percentage points on an annual basis. Figure 2 shows a comparison between three randomly simulated draws of informal assets returns and the formal assets return over a period of 5 years. The figure emphasizes the considerable risk involved in investing in informal assets: the returns vary over a wide range and the may jump from values well above its mean to values close to zero in a matter of months.

Finally, the fifth and the sixth panel present the estimated parameters for the flow income in the unemployment state and for the utility value of working formally. Unemployed individuals are estimated to receive about US$20 per month, which is less than 10% of the average wage. To give a sense of the magnitude of the estimated utility value of working formally, we compared the average flow utility of a formal worker with the one of the average informal worker. Without the added utility benefit of ϵ, the formal worker has a utility 6.8% higher. With the added benefit of ϵ, the formal worker has a utility 9.5% higher. The utility of the additional benefits received by formal workers on top of those that we explicitly parameterize in the model is therefore significant, but it does not constitute the main advantage of working formally.

To provide a further illustration of what our estimated parameters imply, Figure 3 show the steady state distributions by labor market states of total assets, formal financial assets, consumption, and savings. As expected, the distribution of total assets of formal workers first-order stochastically dominates the one of informal workers but a good proportion of both types of workers manage to have positive savings. Most of the unemployed, instead, have to dissave. Consumption level report the same rankings, with unemployed agents consuming significantly less than employed agents.

Table 5 provides an assessment of the in-sample fit of the model by comparing the sample and simulated moments we include in the quadratic form (11). The model fits well all the moments related to proportions in labor market states, wage offers distributions and durations. The estimated model also fits well the portfolio choices, as represented by the proportion of agents keeping more than 50% of their stock of saving in formal assets. We match this statistic both by labor market state and by quartiles of the accepted wage distribution for formal and informal workers. The model fits less well statistics on the actual amount saved. We fit the ranking of the average amount saved (formals save more than informals who in turn save more than the unemployed) but the model underestimate the amount saved by the unemployed by an order of magnitude. This result may be a function of the small sample size: in our sample, only 8% of the unemployed are able to save and the average is driven by a very small number of them saving a relatively high monthly amount.

5 Counterfactual experiments [To Be Completed]

Using the model and the point estimates of the parameters presented in table 3, we perform three sets of counterfactual experiments. In the first, we evaluate the importance of the partial exclusion of informal workers from the formal financial system by giving them full access, i.e. by equating the portfolio costs of informal workers to those of formal workers.[34] In the second, we approximate a technological or policy change that reduces the proportion of job offers that are informal. We reduce the proportion by the amount needed to match the same increase in savings obtained by the full access experiment. The experiment provides a comparison between financial market and labor market policies efficacy in increasing saving. In the last experiment, we evaluate an important tax reform implemented in Colombia in 2012. The reform reduced the payroll contribution that formal workers have to pay from 29.5% to the current 16% (Fernández and Villar, 2017). We "roll-back" the reform by setting $\tau = 0.295$.

In each counterfactual scenario, we evaluate the average impact on labor market outcomes, savings, portfolio decisions, and financial assets, after taking into account the endogenous adjustment in individual's optimal behaviors and in the steady state distributions. We also compute and report the impact of the policy experiments on wealth and consumption inequality. To give a more complete picture, we compute three indices in the class of the Generalized Entropy inequality indices. They are defined as:

$$GE(\alpha) = \begin{cases} \frac{1}{\alpha(\alpha-1)} \left[\frac{1}{N} \sum_{i=1}^{N} \left(\frac{y_i}{\bar{y}} \right)^{\alpha} - 1 \right] & \alpha \neq 0, 1 \\ \frac{1}{N} \sum_{i=1}^{N} \frac{y_i}{\bar{y}} \ln \left(\frac{y_i}{\bar{y}} \right) & \alpha = 1 \\ -\frac{1}{N} \sum_{i=1}^{N} \ln \left(\frac{y_i}{\bar{y}} \right) & \alpha = 0 \end{cases} \tag{12}$$

where N is the number of individuals, y_i is the measure of assets or consumption for individual i and α is a parameter that weights the distance between measurement variable along the distribution. The larger the parameter α, the greater is the weight of the assets/consumption differences among the rich. The three values of the parameter α we report correspond to the following statistics: $GE(0)$ is the mean log deviation, $GE(1)$ is the Theil index, and $GE(2)$ is half the coefficient of variation.

Table 6 reports a set of descriptive statistics in the benchmark model and in the counterfactual simulations. Table 7 reports the inequality indices defined in equation (12). In both Tables, The first column reports the benchmark values, the other six columns reports the policy experiments values. For each experiment, the first column reports the value and the second reports the ratio between the value in the experiment and the benchmark value.

[34] Consistently with equation (3) and table 3's estimate, we set $\psi^e(0) = \psi^e(1) = 0.024$.

5.1 Financial inclusion

The full access experiment (columns 2 and 3) shows that when the portfolio costs are equal for formal and informal workers, savings increase by 3% overall and by 19% for the informally employed. Importantly, the proportion of total assets held by informal workers in formal financial institution increases by more than 80%. As a result, overall formal assets increase by 21%.

Saving behavior also has an impact on labor market decisions since it affects outside options and reservations wages. The proportion of unemployed individuals increases in 0.4 percentage points because a higher wealth makes the individual pickier when evaluating a job offer. This is an example of what are called "unintended consquences" of policy interventions since an increase in unemployment is typically not a policy objective.

Finally, both saving and labor market behaviors have an impact on inequality. Table 7 shows that removing financial exclusions leads to a small decrease in consumption inequality and a larger decrease in the inequality in assets held in formal financial institutions.

5.2 Lower proportion of informal job offers

Results for the experiment reducing the proportion of informal wage offers are reported in columns 4 and 5 of Table 6. The experiment proxies labor market policies able to reduce the proportion of informal job offers, possibly through a combination of enforcement and incentives. We impose a reduction that generates the same increase in savings obtained by the full access experiment (3%). The reduction in $p(0)$ needed is about six percentage points, from the 54.5% to 48.6%. If this experiment is effective in increasing saving and reducing the proportion of informal workers in equilibrium, it is not in increasing the proportion of formal financial assets. In the full access experiment, formal assets increase by 21%, while in this experiment they increase by only 2.3%. In other words, both the labor market policy and the financial market policy are effective in increasing savings but the composition of those savings is very different.

The impact on inequality (Table 7) is weaker than in the financial exclusion experiment: inequality in assets slightly decreases but inequality in consumption does not. Consumption inequality actually increases by up to 1.6%.

5.3 Payroll tax policy

Results for the third experiment are reported in columns 6 and 7 of Table 6. In the experiment we set $\tau = 0.295$, "rolling-back" the 2012 tax reform implemented in Colombia that reduced the payroll contributions for formal workers from 29.5% to 16%. The increase in the payroll tax has a strong impact on the labor market states: the informality rate increases by more than 5 percentage points to 61.5%, while the formality rate drops by a similar magnitude to 34%. This result is quantitatively consistent with what found by Fernández and Villar (2017), who register a drop of 4.8 percentage points in the informality rate after the reform. As a results of this job composition effect, total assets and monthly savings both drop significantly, by about 10%.

Another interesting result is that even with such large change in total wealth, the impact on inequality is almost null (Table 7), which means that the negative effect of the increase in the payroll tax is observed across the whole wealth and consumption distributions.

6 Conclusions [To Be Completed]

We develop and estimate a model able to replicate the crucial features of developing countries economies: the high level of labor market informality, the low level of savings and the high proportion of assets held in informal institutions. We accomplish this result by building a model of labor market search where agents can work formally and informally and can save and borrow in formal and informal financial institutions.

We estimate the model on data for Colombia, a large LAC country with high informality and low saving. The estimates confirm the claim that working informally is linked to saving informally: informal workers face partial financial exclusion from formal financial institutions, as expressed by the higher portfolio costs they face. If full financial access were guaranteed to them, savings would increase 3% a month and formal assets 21%. As a byproduct, asset inequality would decrease 13% and consumption inequality 4%. We also perform a policy experiment to evaluate a recent tax reform implemented in Colombia that significantly reduced payroll contributions for formal workers. We find it had the potential to increase savings by 10% a month.

Our papers also provides two important contributions in the literature of search models of the labor market. First, we extend the literature on search and informality[35] by adding the possibility to save and borrow. Second, we extend the literature on search and saving[36] by adding the option to save in two different types of assets and to work under two different labor contracts.

[35]See Albrecht et al. (2009); Charlot et al. (2013); Bosch and Esteban-Pretel (2012); Meghir et al. (2015); Bobba et al. (2021, 2022).

[36]See Rendon (2006); Lentz (2009); Lise (2013); García-Pérez and Rendon (2020); Abrahams (2022); Danforth (1979); Acemoglu and Shimer (1999); Browning et al. (2007); Krusell et al. (2010); Bils et al. (2011); Ji (2021); Setty and Yedid-Levi (2021); Pizzo (2022).

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Table 1: Descriptive Statistics on Labor Market Outcomes

	Formal Employment	Informal Employment	Unemployment
	Labor Market States		
Proportion	0.395	0.527	0.077
	Wages (hundred of US$ per month)		
Mean	3.284	2.429	–
Standard Deviation	1.395	1.126	–
	Ongoing Duration (months)		
Mean	67.535	89.507	4.034
Standard Deviation	78.689	100.191	6.858
	Sample		
Number Obs.	31709	42307	6195

Table 2: Descriptive Statistics on Saving Behavior

	Formal Employment	Informal Employment	Unemployment
Proportion of Individuals who save			
At all	0.271	0.211	0.036
Mainly in formal institutions	0.493	0.185	0.333
Savings amount among savers (hundred of US$)			
Mean	0.601	0.508	0.443
Standard Deviation	0.721	0.748	0.480
Saving rate among savers (savings/labor income)			
Mean	0.133	0.151	-
Standard Deviation	0.123	0.122	-
Sample Size			
Number Obs.	517	589	83

Table 3: Estimated Parameters

Definition	Parameter	Est. Value	Std. Error
Labor Market Mobility			
Job offer rate - unemployment	λ^u	0.178	(0.0072)
Job offer rate - formal employment	$\lambda^e(1)$	0.034	(0.0054)
Job offer rate - informal employment	$\lambda^e(0)$	0.015	(0.0040)
Job separation rate - formal employment	$\eta(1)$	0.017	(0.0039)
Job separation rate - informal employment	$\eta(0)$	0.014	(0.0027)
Wage Offers			
Proportion of wage offers that are formal	$p(1)$	0.455	(0.0038)
Mean of wages distribution - formal employment	$\mu(1)$	1.056	(0.0519)
Std.Dev. of wages distribution - formal employment	$\sigma(1)$	0.394	(0.0147)
Mean of wages distribution - informal employment	$\mu(0)$	0.800	(0.0369)
Std. Dev. of wages distribution - informal employment	$\sigma(0)$	0.408	(0.0205)
Portfolio Costs			
Cost function parameter - unemployment	ψ^u	0.063	(0.0045)
Cost function parameter - formal employment	$\psi^e(1)$	0.024	(0.0027)
Cost function parameter - informal employment	$\psi^e(0)$	0.224	(0.0314)
Rate of Return for Informal Assets			
Persistence	κ	0.701	(0.0218)
Std. Dev. of the shocks in the rate of return	σ_z	0.037	(0.0006)
Unemployed Income			
Flow income – unemployment	b	0.197	(0.0230)
Utility Value of Labor Market Formality			
Value	ϵ	0.026	(0.0012)
Fixed Parameters			
Relative risk aversion	δ	-0.530	
Payroll tax rate	τ	0.160	
Discount rate	ρ	0.120	
Death rate	θ	0.013	
Rate of return of formal assets	r_1	0.075	
Rate of return of informal assets CI upper limit	$\bar{\bar{r}}_2$	0.158	
Interest rate spread	ν	1.147	
Loss		1.563	

NOTE: Bootstrap standard errors in parentheses.

Table 4: Implied Parameters

Definition	Parameter	Est. Value
Job offers		
Mean of wages distribution - formal employment	$E[w(1)]$	3.106
Std.Dev. of wages distribution - formal employment	$SD[w(1)]$	1.274
Mean of wages distribution - informal employment	$E[w(0)]$	2.418
Std. Dev. of wages distribution - informal employment	$SD[w(0)]$	1.030
Distribution of the rate of return for informal assets		
Mean rate of return of informal assets	$\bar{r}_2$	0.079
Std. Dev. of the rate of return of informal assets	s_{r_2}	0.031

Table 5: Moments Fit

Statistic	Data	Model	Statistic	Data	Model
$e(1)$	0.395	0.394	$E[I_{s>0} \times s \mid e(1)]$	0.163	0.097
$e(2)$	0.527	0.566	$SD[I_{s>0} \times s \mid e(1)]$	0.460	0.223
u	0.077	0.039	$E[I_{s>0} \times s \mid e(0)]$	0.107	0.080
$E[w(1)]$	3.284	3.759	$SD[I_{s>0} \times s \mid e(0)]$	0.400	0.183
$SD[w(1)]$	1.395	1.465	$E[I_{s>0} \times s \mid u]$	0.016	0.001
$E[w(0)]$	2.429	2.854	$SD[I_{s>0} \times s \mid u]$	0.112	0.003
$SD[w(0)]$	1.126	1.153	$E[I_{s>0} \times s \mid e(1), Q_1(w)]$	0.061	0.029
$P5[w(1)]$	2.289	1.790	$E[I_{s>0} \times s \mid e(1), Q_2(w)]$	0.065	0.067
$P5[w(0)]$	0.867	1.348	$E[I_{s>0} \times s \mid e(1), Q_3(w)]$	0.145	0.106
$E[t \mid e(1)]$	5.628	5.950	$E[I_{s>0} \times s \mid e(1), Q_4(w)]$	0.393	0.187
$SD[t \mid e(1)]$	6.557	6.316	$E[I_{s>0} \times s \mid e(0), Q_1(w)]$	0.026	0.029
$E[t \mid e(0)]$	7.459	7.653	$E[I_{s>0} \times s \mid e(0), Q_2(w)]$	0.056	0.051
$SD[t \mid e(0)]$	8.349	8.107	$E[I_{s>0} \times s \mid e(0), Q_3(w)]$	0.096	0.087
$E[t \mid u]$	4.034	4.954	$E[I_{s>0} \times s \mid e(0), Q_4(w)]$	0.310	0.152
$SD[t \mid u]$	6.859	5.922			
$\Pr[\phi > 0.5 \mid e(1)]$	0.493	0.434			
$\Pr[\phi > 0.5 \mid e(0)]$	0.185	0.208			
$\Pr[\phi > 0.5 \mid u]$	0.333	0.314			
$\Pr[\phi > 0.5 \mid e(1), Q_1(w)]$	0.312	0.397			
$\Pr[\phi > 0.5 \mid e(1), Q_2(w)]$	0.458	0.436			
$\Pr[\phi > 0.5 \mid e(1), Q_3(w)]$	0.368	0.450			
$\Pr[\phi > 0.5 \mid e(1), Q_4(w)]$	0.623	0.454			
$\Pr[\phi > 0.5 \mid e(0), Q_1(w)]$	0.000	0.047			
$\Pr[\phi > 0.5 \mid e(0), Q_2(w)]$	0.107	0.176			
$\Pr[\phi > 0.5 \mid e(0), Q_3(w)]$	0.194	0.257			
$\Pr[\phi > 0.5 \mid e(0), Q_4(w)]$	0.353	0.353			

NOTE: $s = da/dt$ is the amount saved, $I_{s>0}$ is an indicator variable that takes the value of 1 if the individual saves a positive amount and zero otherwise, and $Q_i(w)$ represents the quartile i in the observed wages distribution.

Table 6: Counterfactual Experiments - Labor Market and Financial Outcomes

	Benchmark	$\psi^e(0) = \psi^e(1)$		$p(0) = 0.486$		$\tau = 0.295$	
	Value	Value	Ratio	Value	Ratio	Value	Ratio
Labor market states (proportion)							
$e(1)$	0.394	0.393	0.996	0.445	1.129	0.342	0.867
$e(0)$	0.566	0.565	0.997	0.513	0.907	0.615	1.086
u	0.039	0.043	1.077	0.041	1.048	0.043	1.097
Wages (hundred of US$ per month)							
$E[w\|e(1)]$	3.759	3.753	0.999	3.808	1.013	3.772	1.004
$E[w\|e(0)]$	2.854	2.871	1.006	2.870	1.005	2.861	1.003
Total Assets (hundred of US$)							
$E[a]$	6.149	6.365	1.035	6.322	1.028	5.519	0.898
$E[a\|e(1)]$	7.362	7.412	1.007	7.573	1.029	5.768	0.783
$E[a\|e(0)]$	5.495	5.862	1.067	5.499	1.001	5.557	1.011
Formal Financial Assets (hundred of US$)							
$E[\phi a]$	2.241	2.705	1.207	2.305	1.028	1.921	0.857
$E[\phi a\|e(1)]$	3.264	3.223	0.987	3.283	1.006	2.404	0.736
$E[\phi a\|e(0)]$	1.598	2.461	1.540	1.566	0.980	1.704	1.066
Portfolio (proportion of total assets)							
$E[\phi]$	0.310	0.415	1.338	0.317	1.023	0.297	0.957
$E[\phi\|e(1)]$	0.433	0.430	0.994	0.428	0.989	0.401	0.926
$E[\phi\|e(0)]$	0.227	0.415	1.831	0.226	0.998	0.239	1.054
Savings (hundred of US$ per month)							
$E[s\|s > 0]$	0.189	0.195	1.030	0.195	1.030	0.170	0.900
$E[s\|s > 0, e(1)]$	0.221	0.225	1.019	0.226	1.020	0.176	0.797
$E[s\|s > 0, e(0)]$	0.172	0.177	1.030	0.172	1.004	0.170	0.990

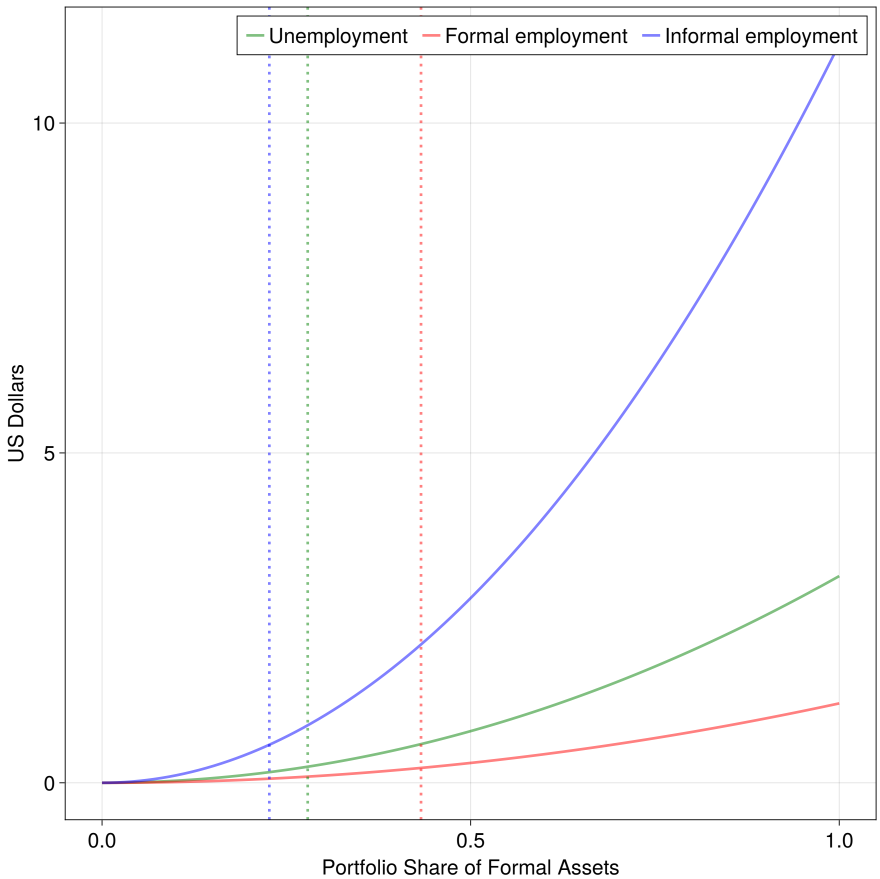
NOTE: Benchmark's values are: $\psi^e(0) = 0.224; \psi^e(1) = 0.024; p(0) = 0.545$; $\tau = 0.160$. $s = da/dt$ is the amount saved. Results are based on simulations of 10.000 individuals. The column Ratio presents the ratio with respect to the benchmark.

Table 7: Counterfactual Experiments - Inequality

	Benchmark	$\psi^e(0) = \psi^e(1)$		$p(0) = 0.486$		$\tau = 0.295$	
	Value	Value	Ratio	Value	Ratio	Value	Ratio
Total Assets							
$GE(0)$	0.277	0.240	0.869	0.270	0.975	0.277	1.001
$GE(1)$	0.224	0.196	0.878	0.220	0.982	0.223	0.997
$GE(2)$	0.247	0.216	0.872	0.242	0.979	0.241	0.975
Financial Assets							
$GE(0)$	0.794	0.359	0.453	0.760	0.956	0.799	1.007
$GE(1)$	0.434	0.232	0.533	0.415	0.955	0.451	1.039
$GE(2)$	1.625	1.135	0.699	1.556	0.958	1.678	1.033
Consumption							
$GE(0)$	0.128	0.126	0.986	0.130	1.016	0.128	1.002
$GE(1)$	0.110	0.107	0.971	0.111	1.007	0.109	0.990
$GE(2)$	0.113	0.108	0.957	0.113	1.002	0.110	0.977

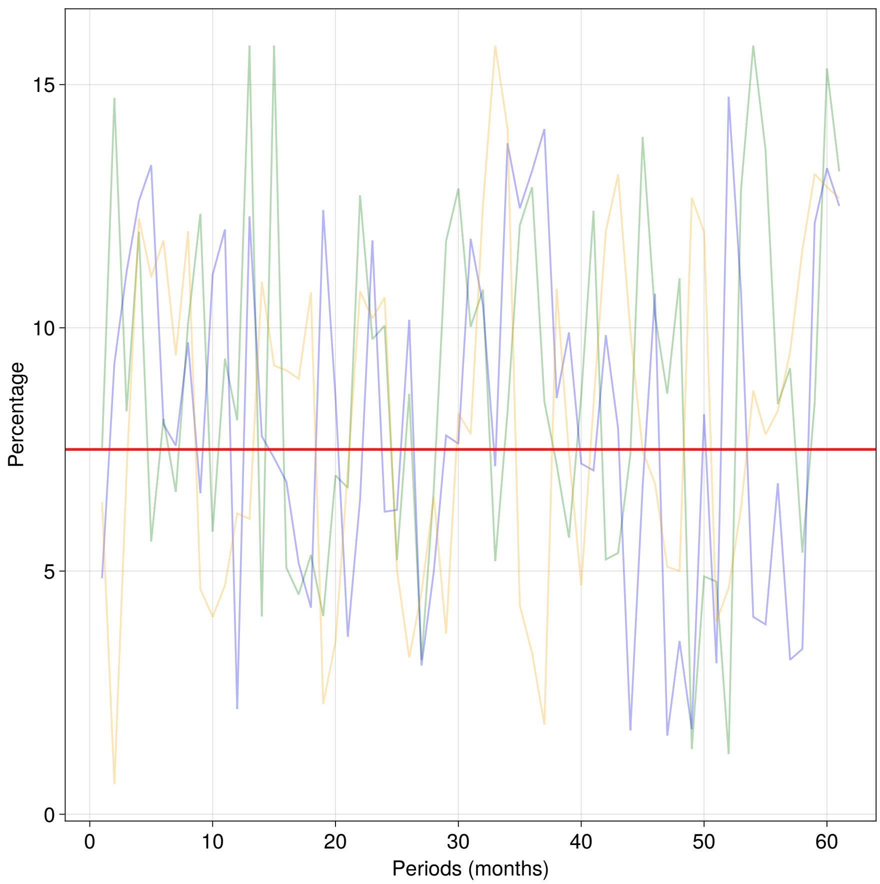
NOTE: $GE(0)$ is the mean log deviation, $GE(1)$ is the Theil index, and $GE(2)$ is half the coefficient of the variation. Benchmark's values are: $\psi^e(0) = 0.224; \psi^e(1) = 0.024; p(0) = 0.545$; $\tau = 0.160$. $s = da/dt$ is the amount saved. Results are based on simulations of 10.000 individuals. The column Ratio presents the ratio with respect to the benchmark.

Figure 1: Cost of Portfolio



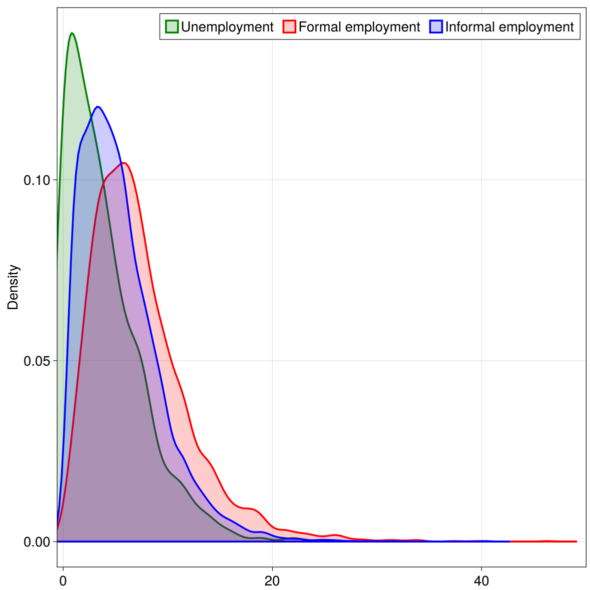
NOTE: Dotted line show the average simulated portfolio allocation by labor market state. Simulated samples of 10,000 individual-level observations based on the estimates reported in Table 3.

Figure 2: Assets Returns

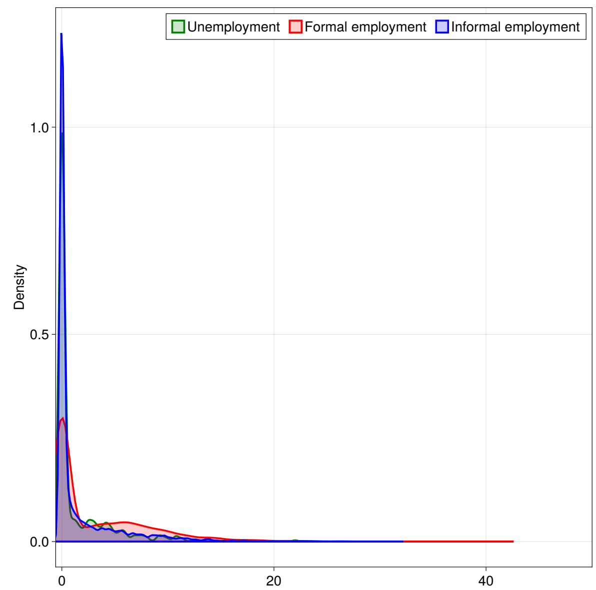


NOTE: Red line show the rate of return of formal (risk-less) assets. Colored lines show simulated samples of 3 individual-level rate of returns of informal assets based on the estimates reported in Table 3.

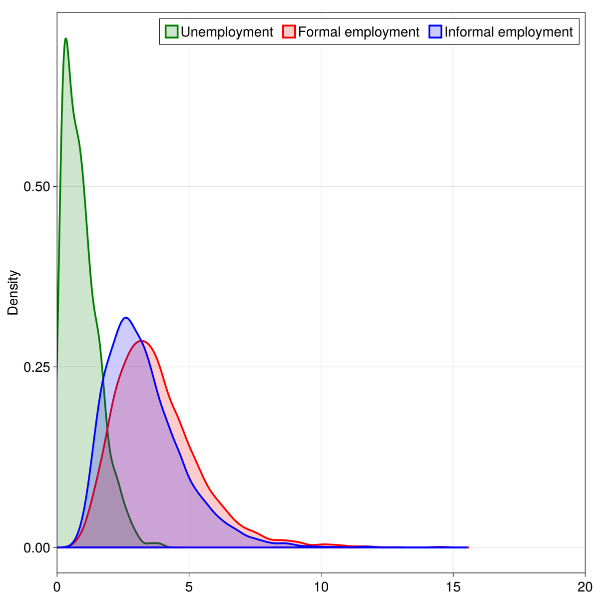
Figure 3: Steady State Distributions of Assets, Consumption and Savings



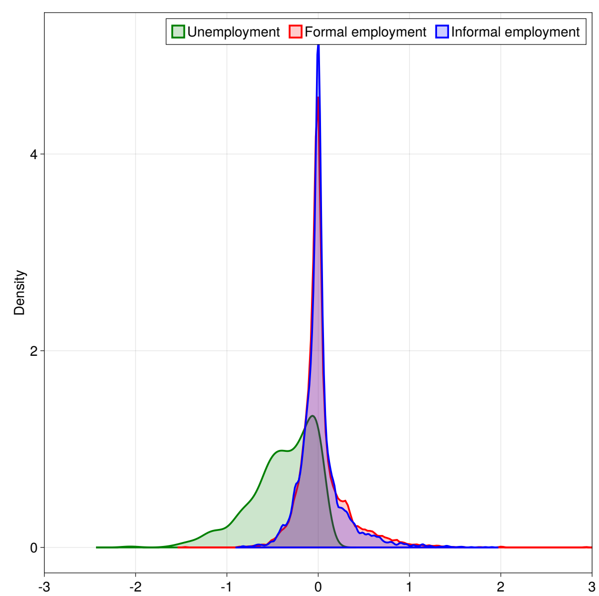
(a) Total Assets



(b) Formal Assets



(c) Consumption



(d) Savings

NOTE: Simulated samples of 10,000 individual-level observations based on the estimates reported in Table 3.

A Model Appendix

A.1 Derivation of the Hamilton-Jacobi-Bellman Equations

The individual problem is:

$$\max \ \mathbb{E}_0 \int_0^\infty e^{-\tilde{\rho}t} \left[u(c) + \epsilon f\right]$$

$$s.to$$

$$dr_2 = \kappa(\bar{r}_2 - r_2)dt + \sigma_z dz$$

$$da = \begin{cases} \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + b - c - \frac{\psi^u}{2}\phi^2\right] dt & \text{if unemployed} \\ \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] dt & \text{if employed} \end{cases}$$

$$a \geq \underline{a} = -\frac{b}{\bar{\bar{r}}_2(1+\nu)}$$

where $dz = \varepsilon_t\sqrt{dt}$ and $\varepsilon_t \sim \mathcal{N}(0,1)$.

The approach we use to derive the Hamilton-Jacobi-Bellman equations of the model is to start with the discrete time version of model, where the length of a time period is Δt, and then take the limit where Δt goes to zero to find the continuous time counterpart. Starting with the case of the employees we have:

$$\begin{aligned} W(a, r_2, w, f) = \max_{0\leq c\leq\bar{c}, 0\leq\phi\leq 1} \Bigg\{ & \left[u(c) + \epsilon f\right]\Delta t + \frac{1}{1+\widetilde{\rho}\Delta t}\mathbb{E}\left[\eta(f)\Delta t U(a_{+\Delta t}, r_{2,+\Delta t})\right. \\ & + \lambda^e(f)\Delta t \sum_{f=0}^{1} \int \max\left\{W(a_{+\Delta t}, r_{2,+\Delta t}, w', f'), W(a_{+\Delta t}, r_{2,+\Delta t}, w, f)\right\} dF(w'|f')p(f') \\ & \left. + \left(1 - \eta(f)\Delta t - \lambda^e(f)\Delta t\right) W(a_{+\Delta t}, r_{2,+\Delta t}, w, f) + o(\Delta t)\right]\Bigg\} \end{aligned}$$

Multiplying by $(1+\widetilde{\rho}\Delta t)$ and rearranging we obtain:

$$\begin{aligned} \widetilde{\rho}\Delta t W(a, r_2, w, f) = \max_{0\leq c\leq\bar{c}, 0\leq\phi\leq 1} \Big\{ & \left[u(c) + \epsilon f\right](1+\widetilde{\rho}\Delta t)\Delta t \\ & + \mathbb{E}\left[\eta(f)\Delta t\left(U(a_{+\Delta t}, r_{2,+\Delta t}) - W(a_{+\Delta t}, r_{2,+\Delta t}, w, f)\right)\right. \\ & + \lambda^e(f)\Delta t \sum_{f=0}^{1} \int \max\left\{W(a_{+\Delta t}, r_{2,+\Delta t}, w', f') - W(a_{+\Delta t}, r_{2,+\Delta t}, w, f), 0\right\} dF(w'|f')p(f') \\ & \left. + \left(W(a_{+\Delta t}, r_{2,+\Delta t}, w, f) - W(a, r_2, w, f, t)\right) + o(\Delta t)\right]\Big\} \end{aligned}$$

Dividing by Δt:

$$\begin{aligned}\widetilde{\rho}W(a,r_2,w,f) = \max_{0\le c\le \bar{c}, 0\le\phi\le 1} & \{[u(c)+\epsilon f](1+\widetilde{\rho}\Delta t) + \mathbb{E}[\eta(f)(U(a_{+\Delta t}, r_{2,+\Delta t}) - W(a_{+\Delta t}, r_{2,+\Delta t}, w, f)) \\ & + \lambda^e(f)\sum_{f=0}^{1}\int \max\{W(a_{+\Delta t}, r_{2,+\Delta t}, w', f') - W(a_{+\Delta t}, r_{2,+\Delta t}, w, f), 0\} dF(w'|f')p(f') \\ & + \left(\frac{W(a_{+\Delta t}, r_{2,+\Delta t}, w, f) - W(a, r_2, w, f, t)}{\Delta t}\right) + \frac{o(\Delta t)}{\Delta t}\Bigg]\Bigg\}\end{aligned}$$

Taking the limit as Δt goes to zero we have $\lim_{\Delta t\to 0}\frac{W(a_{+\Delta t}, r_{2,+\Delta t}, w, f) - W(a,r_2,w,f,t)}{\Delta t} = \frac{dW(a,r_2,w,f)}{dt}$ and $\lim_{\Delta t\to 0}\frac{o(\Delta t)}{\Delta t} = 0$, therefore:

$$\begin{aligned}\widetilde{\rho}W(a,r_2,w,f,t) = \max_{0\le c\le \bar{c}, 0\le\phi\le 1} & \{u(c)+\epsilon f + [\eta(f)(U(a,r_2) - W(a,r_2,w,f)) \\ & + \lambda^e(f)\sum_{f=0}^{1}\int \max\{W(a,r_2,w',f') - W(a,r_2,w,f,),0\} dF(w'|f')p(f') \\ & + \frac{\mathbb{E}[dW(a,r_2,w,f)]}{dt}\Bigg]\Bigg\}\end{aligned}$$

To find the last term of this equation we use the chain rule:

$$\begin{aligned}\frac{dW}{dt} &= \frac{\partial W}{\partial a}\frac{da}{dt} + \frac{\partial W}{\partial r_2}\frac{dr_2}{dt} + \frac{\partial W}{\partial t} \\ dW &= \frac{\partial W}{\partial a}da + \frac{\partial W}{\partial r_2}dr_2 + \frac{\partial W}{\partial t}dt\end{aligned}$$

Taking the second order Taylor expansion of dW we have:

$$\begin{aligned}dW \approx & \frac{\partial W}{\partial a}da + \frac{\partial W}{\partial r_2}dr_2 + \frac{\partial W}{\partial t}dt + \frac{1}{2}\frac{\partial^2 W}{\partial a^2}(da)^2 + \frac{1}{2}\frac{\partial^2 W}{\partial r_2^2}(dr_2)^2 + \frac{1}{2}\frac{\partial^2 W}{\partial t^2}(dt)^2 \\ & + \frac{\partial^2 W}{\partial a \partial t}da\cdot dt + \frac{\partial^2 W}{\partial r_2^2 \partial t}dr_2\cdot dt + \frac{\partial^2 W}{\partial a\partial r_2}da\cdot dr_2\end{aligned}$$

Now using the rules of stochastic calculus, $(dt)^2 = 0$, $dt \cdot dz = 0$, and $(dz)^2 = dt$, we have:

$$\begin{aligned}
dr_2 &= \kappa(\bar{r}_2 - r_2)dt + \sigma_z dz \\
da &= \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] dt \\
(dr_2)^2 &= \kappa^2(\bar{r}_2 - r_2)^2 (dt)^2 + \sigma_z^2 (dz)^2 + 2\sigma_z\kappa(\bar{r}_2 - r_2)dt \cdot dz = \sigma_z^2 dt \\
(da)^2 &= \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right]^2 (dt)^2 = 0 \\
dr_2 \cdot dt &= \kappa(\bar{r}_2 - r_2)(dt)^2 + \sigma_z dz \cdot dt = 0 \\
da \cdot dt &= \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] (dt)^2 = 0 \\
da \cdot dr_2 &= \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] \kappa(\bar{r}_2 - r_2)(dt)^2 \\
&\quad + \left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] \sigma_z dz \cdot dt = 0
\end{aligned}$$

and therefore:

$$\begin{aligned}
dW \approx & \frac{\partial W}{\partial a}\left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] dt \\
& + \left[\frac{\partial W}{\partial r_2}\kappa(\bar{r}_2 - r_2) + \frac{1}{2}\frac{\partial^2 W}{\partial r_2^2}\sigma_z^2 + \frac{\partial W}{\partial t}\right] dt + \frac{\partial W}{\partial r_2}\sigma_z dz
\end{aligned}$$

which is the chain rule in stochastic calculus (Itô's lemma). Finally, given that the value function does not explicitly depend on time $\frac{\partial W}{\partial t} = 0$ and that $\mathbb{E}[dz] = 0$ we find the Hamilton-Jacobi-Bellman equation presented in the main text of the paper:

$$\begin{aligned}
\widetilde{\rho} W(a, r_2, w, f) = & \max_{0 \le c \le \bar{c}, 0 \le \phi \le 1} \{u(c) + \epsilon f + [\eta(f)(U(a, r_2) - W(a, r_2, w, f)) \\
& + \lambda^e(f) \sum_{f=0}^{1} \int \max\left\{ W(a, r_2, w', f') - W(a, r_2, w, f,), 0\right\} dF(w'|f')p(f') \\
& \frac{\partial W}{\partial a}\left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + w(f)(1-\tau f) - c - \frac{\psi^e(f)}{2}\phi^2\right] \\
& + \frac{\partial W}{\partial r_2}\kappa(\bar{r}_2 - r_2) + \frac{1}{2}\frac{\partial^2 W}{\partial r_2^2}\sigma_z^2\Big]\Big\} \qquad (13)
\end{aligned}$$

To derive the Hamilton-Jacobi-Bellman equation for the unemployment state we proceed in the same fashion. The discrete time version where the length of a time period is Δt is:

$$U(a,r_2) = \max_{0\le c\le \bar{c}, 0\le \phi\le 1} \{u(c)\Delta t + \frac{1}{1+\widetilde{\rho}\Delta t}\mathbb{E}\left[\lambda^u \Delta t \sum_{f=0}^{1} \int \max\{W(a_{+\Delta t}, r_{2,+\Delta t}, w, f), U(a_{+\Delta t}, r_{2,+\Delta t})\}\, dF(w|f)p(f) + (1-\lambda^u\Delta t)U(a_{+\Delta t}, r_{2,+\Delta t}) + o(\Delta t)\right]$$

Rearranging, dividing by Δt and taking the limit as Δt goes to zero we have:

$$\widetilde{\rho}U(a,r_2) = \max_{0\le c\le \bar{c}, 0\le \phi\le 1} \left\{u(c) + \mathbb{E}\left[\lambda^u \sum_{f=0}^{1} \int \max\left\{\ W(a,r_2,w,f)\ \ - U(a,r_2), 0\right\} dF(w|f)p(f) + \frac{\mathbb{E}\left[dU(a,r_2)\right]}{dt}\right]\right.$$

Finally, using again Itô's lemma we obtain the Hamilton-Jacobi-Bellman equation presented in the main text of the paper for the unemployment state:

$$\tilde{\rho}U(a,r_2) = \max_{0\le c\le \bar{c}, 0\le \phi\le 1} \left\{u(c) + \lambda^u \sum_{f=0}^{1}\left(\int_w \max\{W(a,r_2,w,f) - U(a,r_2), 0\} dF(w|f)p(f)\right) \frac{\partial U}{\partial a}\left[(r_1\phi + r_2(1-\phi))(1+\nu I_{a^-})a + b - c - \frac{\psi^u}{2}\phi^2\right] + \frac{\partial U}{\partial r_2}\kappa(\bar{r}_2 - r_2) + \frac{1}{2}\frac{\partial^2 U}{\partial r_2^2}\sigma_z^2\right\} \tag{14}$$

A.2 Solution Method

We use a two-step approach to solve the model. In the first step, we solve the Hamilton-Jacobi-Bellman equations using a value function iteration method. Following Achdou et al. (2014, 2017), the derivatives of the value functions are approximated using a finite difference with an upwind scheme. That is, a forward difference is used whenever the drift of the state variable (here, $da > 0$ or $dr_2 > 0$) is positive. On the contrary, the backward difference is used when a negative drift occurs. According of Barles and Souganidis (1991), if the conditions of monotonicity, stability, and consistency hold, the solution of the Hamilton-Jacobi-Bellman equations is unique. As Achdou et al. (2017) argues, stability and consistency conditions are relatively easy to hold in this type of model, however for the monotonicity condition to be satisfied the upwind scheme is crucial. An additional advantage of the upwind finite difference scheme is that it allows for the handling of optimization state constraints in a very convenient way. In particular, if the boundary condition associated with the borrowing constraint $a \ge \underline{a}$ is set for the backward difference, and not for the forward difference, and the upwind scheme is allowed to choose the right difference, according to the drift at the bottom of the assets distribution, it will be possible to guarantee that the borrowing constraint is never violated.

In the second step we solve for the invariant distributions of labor market states and of total assets. For this highly non linear model, the transition equations for the state distributions (i.e. the Kolmogorov Forward equations) do not have an obvious explicit algebraic representation and, therefore, cannot be used to compute the invariant distributions. Instead, we use a simulation approach. We simulate labor market careers for a large number of individuals – starting everyone as unemployed with zero assets – and for a large number of periods until both the distributions of labor market states and total assets stabilize. In each period of the career we use the value functions of the first step, together with the optimal decision rules, to govern individual choices on job offers, portfolio allocation and consumption/savings. In the computations we simulate 10,000 careers and obtain the invariant distributions after 800 model periods.

B Identification Appendix

Hazard rates out of the three labor market states.

Unemployment:

$$h_U(a, r_2) = \lambda^u \sum_{f=0}^{1} p(f)[1 - F(\tilde{w}(a, r_2, f)|f)] \tag{15}$$

$$h_U = \int\int h_U(a, r_2) d\Lambda(a) d\Gamma(r_2) \tag{16}$$

$$E[t_U] = \frac{1}{h_U} \tag{17}$$

The sample analog of equation (17) identifies λ^u, given that, under the model, $\Lambda(a)$, $\Gamma(r_2)$ and $F(w|f)$ are mainly identified by accepted wages and savings. Notice that we have to integrate out both the steady state distribution of assets and the stochastic return to the risky asset because none of them is observable to us. Therefore, we only observe durations for all the unemployed individuals. Their sample mean is the sample analog of the LHS of (17).

Employment for $f = 0, 1$:

$$h_E(a, r_2, w, f) = \eta(f) + \sum_{f'=0}^{1} \lambda^e(f) p(f')[1 - F(\hat{w}(a, r_2, w, f, f')|f)] \quad (18)$$

$$h_E(a, r_2, f) = \int_{\tilde{w}(a,r_2,f)} h_E(a, r_2, w, f) \frac{f(w|f)}{1 - F(\tilde{w}(a, r_2, f)|f)} dw \quad (19)$$

$$h_E(f) = \int\int\int_{\tilde{w}(a,r_2,f)} h_E(a, r_2, w, f) \frac{f(w|f)}{1 - F(\tilde{w}(a, r_2, f)|f)} dw d\Lambda(a) d\Gamma(r_2) \quad (20)$$

$$E[t_E(f)] = \frac{1}{h_E(f)} \quad (21)$$

Equation (21) defines two equations, one for $f = 0$ and one for $f = 1$. Again leveraging on the constrained imposed by the model and the identification provided by savings and accepted wages, the sample analog of $E[t_E(0)]$ jointly identifies $\{\eta(0), \lambda^e(0)\}$ while the sample analog of $E[t_E(1)]$ jointly identifies $\{\eta(1), \lambda^e(1)\}$. To separately identify them, we add information from the steady state probability in each state. Using the law of motion for the measure of unemployed individuals v and applying the steady state condition $\dot{v} = 0$, we find:

$$v = \frac{\sum_{f=0}^{1} \eta(f) e(f)}{h_U} \quad (22)$$

which jointly identifies $\{\eta(0), \eta(1)\}$. By adding the two steady state expressions for $\dot{e}(f) = 0$, we obtain the two additional equations to separately identify the four mobility parameters. We need two additional equations because of the constraint: $v + e(0) + e(1) = 1$.