Dual labor markets and labor protection in an estimated search and matching model

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1. Introduction

Temporary contracts are widely used in European countries and, in the last twenty years, have proliferated in developing nations, particularly in Latin American countries (Harrison and Leamer, 1997; Heckman and Pages, 2000). They have been used to introduce flexibility in the labor market in order to reduce unemployment. However, the literature has found an ambiguous effect of temporary contracts on unemployment because these contracts do not only affect the flows out of unemployment but also the flows out of employment for newly hired workers (see for example, Bentolila and Dolado, 1994; Blanchard and Landier, 2002; Güell, 2003; Aguirregabiria and Alonso-Borrego, 2009, among others). At the same time, labor protection, in the form of firing costs, has also been extensively used to reduce unemployment with the difference that this policy leads to fewer job destructions. The literature has also found that labor protection affects the job creation rate, generating an ambiguous effect on unemployment (see for example, Mortensen and Pissarides, 1994, 1999b, among others).

A large part of the literature that analyzes both temporary contracts and labor protection policies has treated the proportion of available temporary contracts in the economy as exogenous. Little attention has been given to the interaction between these two policies when that proportion is an outcome of the equilibrium in the labor market. In this line of the literature, the closest paper to this research is Cahuc and Postel-Vinay (2002). However, data for some OECD and Latin American countries suggest that temporary contracts are in fact used to reintroduce flexibility when firing costs are high. In a way, this implies that employers try to avoid firing restrictions by replacing permanent workers with temporary workers (Harrison and Leamer, 1997). This idea is captured in Fig. 1 which shows a positive relation between the degree of protection of permanent jobs and the employment rate of temporary contracts observed in the economy. Hence, the following question arises: Once the government authorizes the use of temporary contracts, are these contracts an equilibrium response of firms to introduce flexibility when firing costs are high? An interest in this endogenous relation has only recently emerged, and the related literature is still scarce (see Cao et al., 2011; Alvarez and Veracierto, 2012; Macho-Stadler et al., 2011; Paolini and de Tena, 2012).

In addition to the policy implications of temporary contracts, there are concerns regarding the use of these contracts in Latin American countries since they represent a phenomenon of job insecurity (like informality) and can potentially have important effects on productivity and growth. In particular, they are associated with lower investment in human capital and productivity losses because the lack of attachment to the firm reduces the incentives of firms to invest in workers.

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2 There is an exception that occurs in the absence of perfect insurance markets where employment protection (chosen optimally) plays a role of insurance and the job creation rate is not affected (Pissarides, 2001).

3 The degree of protection is captured by the index constructed by Pierre and Scarpetta (2004).

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Therefore, following this line of the literature, temporary contracts workers if the probability of becoming a permanent worker is low. Temporary jobs a\footnote{Dolado and Stucchi (2008) also analysis of the Chilean labor market, Pierre and Scarpetta (2004), Tokman and Martinez (1999), OECD Stats.}.

This paper tries to answer the following questions. First, given the existence of both permanent and temporary contracts, and the potential trade-off between flexibility and productivity related to them, does more protection to permanent jobs generates a greater proportion of available temporary contract in the economy, increasing with that the employment rate of temporary jobs\footnote{ff} Second, is labor protection effective to reduce unemployment when permanent and temporary jobs coexist? Finally, in dual labor markets (that is, with permanent and temporary contracts), are agents better off and labor market outcomes less unequal? To answer these questions, this paper extends Mortensen and Pissarides\footnote{(Heckman and Pages, 2000). Empirically, Carpio et al. (2011), in their analysis of the Chilean labor market, find that having a temporary contract reduces the probability of receiving employer-paid training. Dolado and Stucchi (2008) also find that, in the case of Spain, temporary jobs affect total productivity; however, their mechanism is different. In their study, temporary contracts reduce the effort of workers if the probability of becoming a permanent worker is low. Therefore, following this line of the literature, temporary contracts generate a trade-off between flexibility and productivity gains.}.

The proposed model also includes factual exercises to quantitatively evaluate the role of labor protection\footnote{This paper makes the following assumptions about labor protection. First, a temporary contract is broadly defined as any contract with a specified duration. (2) Temporary contracts are important regardless of worker age (the share of these contracts is roughly constant across age groups), (3) Temporary contracts last on average less than permanent contracts but more than 12 months (over 100 months for permanent contracts vs. close to 40 months for temporary contracts), (4) workers with permanent contracts earn more on average (almost 60% more), but there are also workers with temporary contracts earning high wages, (5) temporary contracts have a higher prevalence in some sectors, particularly among the unskilled workers (such as construction, agriculture and mining).}.

Additionally, under the definition of temporary contracts, it is still possible to have overlapping employment of contingent workers, and that employment is one of the lowest for Latin American standards -less than 20% of employment is in the informal sector \footnote{Second, aside from Brazil, Chile is one of the countries with the highest and more persistent levels of income inequality, not only at the regional level but also worldwide. Third, the level of informality in the Chilean labor market is one of the lowest for Latin American standards, with only 13% of temporary contracts, and in the case of the Mexican labor market, the proportion of temporary workers is even lower, 9.3%}. The inclusion of these last contingent job arrangements in the definition of temporary contracts is relevant for Latin American countries, where a large proportion of the labor force is involved in agriculture and other primary activities making fixed-term contracts less relevant than other types of temporary contracts (particularly seasonal jobs). Second, as in Cahn and Postel-Vinay\footnote{(2002), it is assumed that there are two types of jobs that coexist in the market, permanent and temporary. Additionally, because the type of contract can influence the productivity of the job (and production opportunities, particularly for seasonal jobs), it is assumed that the productivity distributions are contract specific. This last assumption also allows for the fitting of overlapped productivities and wages distributions, as is the case of the Chilean labor market. Indeed, as Kalleberg (2000) suggests, temporary workers earn, on average, lower wages. However, since there is considerable heterogeneity in the wages for temporary workers, it is possible to find jobs in which temporary worker earnings are higher than those of regular employees. Lastly, because temporary contracts have a pre-defined duration, the decision rules they generate have a non-stationary feature. Additionally, under the definition of temporary contracts.}.
used in this paper, it is possible to have different contract lengths for different temporary jobs, making it necessary to follow a distribution of those durations. These two features greatly complicate both the theoretical and empirical analysis because it is necessary to follow a distribution of non-stationary decision rules. To avoid this complication, this paper follows Wasmer (1999) in choosing a shortcut and modelling temporary contracts as regular jobs but interpreting the inverse of the hazard rate out of temporary jobs as the average contract length associated with temporary contracts.\(^{10}\)

In the estimation process there are observed heterogeneity controls because there is evidence that the wage gap between temporary and permanent jobs depends on education and gender (Felgueroso and De la Rica, 1999). In particular, the sample used in this paper is comprised of unskilled male workers (without a college degree), since the higher wage gaps are usually found in the bottom of the distribution (Bosio, 2009).

The results obtained show that the availability of temporary contract jobs increases and its share in the labor market becomes more important as labor protection of regular jobs increases. This result is consistent with the idea that firms use this type of contracts to reintroduce flexibility when there is an increase in firing costs. Moreover, the existence of temporary contracts offset the positive effect of labor protection on unemployment and therefore labor protection generates a substitution effect between contracts. Additionally, inequality is very persistent and labor protection is not an effective labor market policy in this respect. Finally, temporary contracts increase flexibility and make workers and firms better off. Welfare gains from temporary contracts become larger as labor protection gets very stringent. This is a steady-state result and it could be magnified if any cushion effect on business cycles is taken into account.

The paper is organized in the following manner. Section 2 presents the search and matching model and defines its steady-state equilibrium. Section 3 describes the data used in the estimations and the procedure followed to obtain the estimation sample. Section 4 presents the estimation method, discusses the identification strategy, and reports the estimation results. Section 5 contains the counterfactual and policy experiments, and finally, Section 6 concludes.

2. The model

This section describes the model setup and the determination of the steady state equilibrium. The model is an extension of Mortensen and Pissarides (1994) in which it is assumed that both temporary and permanent contracts coexist and its availability in the market is determined endogenously as part of the equilibrium.\(^1^{1}\). It is assumed that time is continuous and that the economy is populated by infinitely lived workers, who are risk neutral and ex-ante homogenous. There is also a continuum of firms that produce their output with a fixed-coefficient technology using only labor as input. In addition, it is assumed that the labor market environment is stationary and that the search process is random. Search frictions are characterized by a matching function, which depends on the overall market tightness and the proportion of available vacancies in each type of contract. The model further assumes that there are two invariant worker-firm productivity distributions: one for each type of contract. Once a firm meets a worker, a match-specific productivity, conditional on the type of contract, is drawn from the relevant productivity distribution, previously mentioned, and wages are determined by Nash bargaining.

Only unemployed workers search for jobs, that is, there is no on-the-job search in the model.

The main differences between permanent and temporary contracts are due to employment protection legislation and productivity gains. On one hand, permanent contracts are related to regular jobs, for which there is no specified term in the contract and there is job protection in the form of firing costs. On the other hand, workers with permanent contracts are subject to idiosyncratic productivity shocks, which can be positive or negative. Positive shocks are interpreted as productivity gains,\(^1^{2}\), while negative shocks can lead to a destruction of the worker-firm match, implying that permanent contracts are subject to endogenous job destruction.

Additionally, temporary contracts are broadly defined as contracts characterized by a defined-duration and for which the duration is specific to the contract. This implies that these contracts can last a finite number of periods (possibly more than a year) and two contracts can differ in their durations. These two features generate non-stationarity in the decision rules and introduce heterogeneity in contract durations, which complicate considerably the theoretical and empirical analysis. To avoid these complications, the modelling strategy follows Wasmer (1999) and takes a shortcut assuming that temporary contract jobs are subject to exogenous termination shocks, and that its average duration is interpreted as that of the temporary contracts in the economy. In particular, temporary contracts can be terminated, at no cost, because the maximum duration expires or due to a destruction shock; both cases are treated as an exogenous termination. Finally, it is assumed that, in some cases, firms can transform this type of contract into a permanent contract upon termination. Because both types of jobs are technologically different, the transition from a temporary to a permanent contract is interpreted as an occurrence of a reallocation shock.

The introduction of firing costs, in the form of severance tax payments, has important implications on wage determination, since firing costs change the threat point in the Nash bargaining game. In particular, if a firm with a permanent contract meets a worker, then they bilaterally bargain the wage; if the job is not created (due to a bad productivity draw) then both the worker and the firm continue the searching process without any severance tax payment. However, if a worker is currently employed with a permanent contract and he receives a productivity shock, then the worker and the firm engage in a wage renegotiation process. In this case, the firm has to pay the severance tax if the job is destroyed; therefore, the bargaining position of the worker is better (the outside option of the firm is different in both cases). Following the same terminology as in Pissarides (2000), a newly hired worker is called an outsider worker, while a continuing employee is called an insider worker. Additionally, payroll taxes exist on both sides of the market. As in Albrecht et al. (2009), and to simplify the analysis, it is assumed that the collected taxes, both payroll and severance, are not redistributed among workers and are just thrown into the ocean.

It is important to mention that the distinction between the two types of contracts is related to the degree of flexibility and not to the degree of formality (or informality) of the labor market. Both types of contracts are related to formal jobs.\(^1^{13}\).

\(^{10}\) In the case of fixed-term contracts, the hazard rate should produce an average duration of 12 months.

\(^{11}\) Aside from being the standard tool in the literature for labor market policy analysis (Albrecht et al., 2009), the model in Mortensen and Pissarides (1994) studies the interaction between labor market protection and endogenous job destruction, which is particularly relevant for the analysis in this paper.

\(^{12}\) In particular, these productivity gains can be interpreted as human capital investments, which exist in this type of contract given the incentives provided by a permanent contractual relation (Heckman and Pages, 2000).

\(^{13}\) Even though informality is not considered in the model, information about potentially informal workers is not dropped in the estimation and its labor market outcomes are considered for estimation purposes. In terms of magnitude, if informality is defined as if the worker has a signed contract or not, less than 9% of the workers in the sample are considered informal workers (more than 60% of these workers are in temporary jobs).
2.1. Workers’ value functions

At any point in time, workers can be in any one of the following four states: unemployed, employed as a new hire with a permanent contract (indexed by OP, outsider permanent), a continuing employee with a permanent contract (indexed by IP, insider permanent), and employed as a new hire with a temporary contract (indexed by T). Let $u$ be the rate of unemployment, and $v_1$ and $v_2$ be the job vacancy rates with respect to permanent and temporary contracts, respectively. Therefore, the total vacancy rate is $v = v_1 + v_2$. If the population is normalized to 1, then the rates at which workers and firms potentially match are $m(u, v_1)$ and $m(u, v_2)$, respectively; $m(·)$ is the matching function, which is increasing in both arguments, concave and homogeneous of degree one. Defining the overall labor market tightness as $q = \frac{v_1 + v_2}{v_1 v_2}$ and the proportion of permanent contract vacancies with respect to the total number of vacancies as $q_1 = \frac{v_1}{v_1 + v_2}$, and using the homogeneity property of the matching function, it is possible to write the rates at which unemployed workers meet job vacancies with permanent and temporary contracts as $a_1 = \frac{vm(u, v_1)}{v_1 v_2} = m[v(q_1)]$ and $a_2 = \frac{vm(u, v_2)}{v_1 v_2} = m[1 - \eta q_1]$, respectively. It is assumed that only unemployed workers search for a job (there is no on-the-job search).

When a worker meets a vacancy, there is a match-specific productivity. Let $f(x)$ be the invariant worker-firm productivity distribution, for $i = P, T$, from which the productivity $x$ is drawn. Not all meetings create a job because not all workers draw a high enough productivity to make the match worthwhile. Only draws higher than the reservation productivity of new hires under permanent contracts ($x_{DOP}$) or higher than the reservation productivity under temporary contracts ($x_{DT}$) end up with a job creation for each type of contract. Let $U$ be the value of unemployment, $W_{OP}(x)$ be the value of employment for a new hire under a permanent contract, and $W_T(x)$ be the value of employment for a worker hired under a temporary contract. Therefore, the flow value of unemployment is expressed as:

$$rU = b + \alpha_P \int_0^\infty [W_{OP}(x) - U]f_P(x)dx$$

$$+ a_1 \int_0^\infty [W_T(x) - U]f_T(x)dx$$

(1)

While unemployed, individuals receive a utility (or disutility) $b$ interpreted as the flow income, which is equivalent to the value of leisure. At rate $\alpha_P$, a worker meets a vacancy with a permanent contract, and if a job is created then there is a capital gain of $W_{OP}(x) - U$. Similarly, a worker meets a vacancy with a temporary contract, at rate $\alpha_T$, and when the job opportunity is taken there is a capital gain of $W_T(x) - U$. In order to write the flow value of employment under a permanent contract, and according to the previous discussion regarding the effect of firing costs on wages, it is necessary to distinguish between a new hire (outsider) and a continuing employee (insider), both under permanent contracts. Let $W_{OP}(x)$ and $x_{DOP}$ be the value of employment and the reservation productivity, respectively, for a continuing employee under a permanent contract. The flow value of an outsider worker with a permanent contract and current productivity $x$ can then be written as:

$$rW_{OP}(x) = w_{OP}(x)(1 - \tau_p) + \lambda_P \int_0^\infty w_{OP}(x')f_{OP}(x')dx'$$

$$+ \lambda_P W_{OP}(x)U - \lambda_P W_{OP}(x)$$

(2)

while the flow value of an insider worker with a permanent contract can be expressed as:

$$rW_{IP}(x) = w_{IP}(x)(1 - \tau_p) + \lambda_P \int_0^\infty w_{IP}(x')f_{IP}(x')dx'$$

$$+ \lambda_P W_{IP}(x)U - \lambda_P W_{IP}(x)$$

(3)

An outsider worker with a permanent contract in a job with productivity $x$ receives an after payroll tax wage rate of $w_{OP}(x)(1 - \tau_p)$. A productivity shock arrives at a Poisson rate $\lambda_P$. If the new productivity $x'$ is above the reservation productivity $x_{DOP}$, then the worker remains employed, but he is now an insider worker with a capital gain or loss of $w_{OP}(x')(x' - \tau_P)$. There is the possibility of productivity gains if $x' > x$. On the contrary, if the new productivity is below the reservation productivity, then the worker becomes unemployed and the capital loss is $w_{OP}(x') - W_{OP}(x')$. If an insider worker with a permanent contract continues as an employee, then he receives an after-payroll-tax wage rate of $w_{IP}(x)(1 - \tau_p)$ and a capital gain or loss of $W_{IP}(x') - W_{IP}(x)$; but if the job is terminated, then the capital loss for the worker is $U - W_{OP}(x)$.

When a worker is employed with a temporary contract, the flow value is:

$$rW_T(x) = w_T(x)(1 - \tau_T) + \lambda_T \int_0^\infty w_T(x')f_T(x')dx'$$

$$+ [\delta_T + \lambda_T W_{IP}(x)U] - \lambda_T W_T(x)$$

(4)

In this case, a worker with a temporary contract and productivity $x$ receives an after-payroll-tax wage rate of $w_T(x)(1 - \tau_T)$. He loses his job and becomes unemployed at a Poisson rate $\lambda_T$ with a consequent capital loss of $U - W_T(x)$. Finally, at Poisson rate $\delta_T$ his job is terminated but instead of going directly to unemployment, the worker has the possibility to immediately meet a firm with a permanent contract and draw a new productivity. If that productivity is high enough (higher than the reservation productivity $x_{DOP}$), then the worker becomes a new hire with a permanent contract and generates a capital gain of $W_{OP}(x) - U$, otherwise he becomes unemployed and starts the searching process again.

2.2. Firms’ value functions

$I_{OP}(x)$ and $I_{IP}(x)$ are defined as the values of a filled job for a new hire (outsider) and a continuing employee (insider), both under permanent contracts, respectively. Similarly, $I_T(x)$ is defined as the value of a filled job under a temporary contract. Also, let $V_P$ and $V_T$ be the values of creating a vacancy for each type of contract, permanent and temporary respectively. Using these definitions, the flow values of a filled job under a permanent contract can be written as:

$$rI_{OP}(x) = x - w_{OP}(x)(1 + \phi_P) + \lambda_P F_{OP}x_{DOP}[V_P - I_{OP}(x) - \mathcal{Y}]$$

$$+ \lambda_P \int_0^\infty [I_{OP}(x') - I_{OP}(x)]f_{OP}(x')dx'$$

(5)

and

$$rI_{IP}(x) = x - w_{IP}(x)(1 + \phi_P) + \lambda_T F_{IP}(x_{DOP})[V_P - I_{OP}(x) - \mathcal{Y}]$$

$$+ \lambda_T \int_0^\infty [I_{IP}(x') - I_{IP}(x)]f_{IP}(x')dx'$$

(6)

Firms using permanent contracts receive a flow output of $x$ and pay an after payroll tax wage rate of $w_{OP}(x)(1 + \phi_P)$ if a worker is an outsider, and $w_{IP}(x)(1 + \phi_P)$ if he is an insider. In this setup, both the employer and the employee pay payroll taxes, making it possible to

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15 The assumption that transitions from temporary to permanent jobs occur upon termination is supported by the data, because when workers are asked to provide the reason for termination of the current (temporary) job, more than 85% of those who changed to a permanent job stated that the contract expired or that they were fired. Moreover, less than 10% explicitly stated that they left their current job for a better job.

16 Even though the reallocation shock is introduced in the model to account for those temporary jobs that are converted to permanent jobs, it is assumed that the worker draws a new productivity because it is not possible to identify (in the data) which of the job-to-job transitions occur with the same employer. Hence, the reallocation shock captures any transition from a temporary to a permanent job.
differentiate the source of the tax payment. There are two possible outcomes when there is a productivity shock. First, for any productivity greater than the reservation productivity the firm continues producing and the capital gains or losses are \( I_0(x^*) - I_0(x) \) and \( I_0(x^*) - I_0(x) \) for an outsider and an insider worker, respectively. Second, if the shock is sufficiently bad, that is, the new productivity has fallen below the reservation productivity, then the worker is dismissed and the firm has to pay the severance tax \( \Psi \). In this case, the capital loss, \( V_f - J(x) - \Psi \) for \( i = IP, OP \), takes into account that the firm now has an unfilled vacancy and has to pay the severance tax.

In turn, the flow value of a filled job under a temporary contract is:

\[
r_f(x) = x - w_f(x)(1 + \phi_f) + (\lambda_f + \lambda_g)(V_f - J(x)) \tag{7}
\]

Firms using temporary contracts receive a flow output of \( x \) and pay an after payroll tax wage rate of \( w_f(x)(1 + \phi_f) \). It is assumed that the payroll taxes differ with the types of contracts. When there is either a termination shock or a reallocation shock the match is destroyed at no cost, generating a capital loss of \( V_f - J(x) \) to the firm. Thus, the firm engages once again in the search process.

Finally, the flow values of unfilled vacancies for both types of contracts are:

\[
r_{VP} = -k_P + a_P \int_0^\infty [I_{OP}(x) - V_f]f_p(x)dx \tag{8}
\]

\[
r_{VP} = -k_T + a_T \int_0^\infty [I_{OP}(x) - V_f]f_p(x)dx \tag{9}
\]

To keep the vacancies while searching, firms pay a per-period fixed cost of \( k_P \) and \( k_T \), according to the type of contract, permanent and temporary, respectively. At rate \( a_T \), firms with a permanent contract job meet workers, and if the realized match-specific productivity is good enough (greater than the reservation productivity \( \tau^{v*}_T \) of a new hire with this type of contract), then the vacancy is filled and the firms have a capital gain of \( I_0(x^*) - V_f \). In the case of firms with a temporary contract job, meetings occur at rate \( a_T = \frac{\lambda_T}{1 - \tau_T} \) and the capital gain for the firms is \( J(x) - V_f \) if the job is created.

2.3. Steady-state equilibrium

The steady-state condition requires that both: (1) the flow into jobs with permanent contracts, coming either from unemployment or temporary jobs, is equal to the flow out of permanent contract jobs:

\[
a_T^p [1 - F_f(x_T^*)]u + a_T^T [1 - F_f(x_T^*)]e_T = \lambda_T F_f(x_T^*)e_f
\]

and (2) the flow into jobs with temporary contracts is equal to the flow out of these jobs:

\[
a_T^p [1 - F_f(x_T^*)]u = (\lambda_T + \lambda_g) e_T
\]

Combining the last two equations and using the fact that \( e_f + e_T + u = 1 \) makes it possible to find expressions for the unemployment rate (the Beveridge curve) and the employment rates in temporary and permanent contract jobs. To reduce notation lets define the hazard rate out of the state \( i \) and going to the state \( j (h_i) \) as:

\[
h_{TP} = a_T^p [1 - F_f(x_T^*)]
\]

\[
h_{TP} = a_T^T [1 - F_f(x_T^*)]
\]

\[
h_{TT} = \lambda_T
\]

\[
h_{TT} = \lambda_T + \lambda_g
\]

and therefore:

\[
u = \frac{h_{TT}h_T}{h_{TP}h_T + h_{PT}h_T + h_{TT}h_T} \tag{10}
\]

\[
e_T = \frac{h_{TP}h_T + h_{TT}h_T}{h_{TP}h_T + h_{PT}h_T + h_{TT}h_T} \tag{11}
\]

\[
e_T = \frac{h_{TP}h_T + h_{TT}h_T}{h_{TP}h_T + h_{PT}h_T + h_{TT}h_T} \tag{12}
\]

The next step in finding the equilibrium is defining how wages are determined. Since workers and employers meet on a bilateral basis, wages are determined in a bargaining process between both parties once the match-specific productivity is realized. As is a common practice in the literature, the generalized axiomatic Nash bilateral bargaining outcome is used to determine wages (Mortensen and Pissarides, 1999a). If \( \beta_F \) and \( \beta_F \) are the worker's relative bargaining power parameters when he faces an employer offering a permanent and a temporary contract, respectively, the different wage rates solve the following optimization problems (according to the type of contract and if the worker is an outsider or an insider):\(^{17}\):

\[
\max\left(W_{OP}(x) - U\right)^\beta_P (I_0(x) - V_f)^{\gamma_P} \tag{13}
\]

\[
\max\left(W_{OP}(x) - U\right)^\beta_P (I_0(x) - V_f + \Psi)^{\gamma_P} \tag{14}
\]

From the worker’s point of view, the threat point is simply the value of breaking the contract, which is the value of unemployment. From the firm’s point of view, the threat point is the value of continued search, and it differs depending on the type of contract and whether the worker is an outsider or an insider. If an unemployed worker meets a firm with a permanent contract (the worker becomes a new hire or an outsider if the job is formed), then the threat point in the bargaining process is the value of an unfilled vacancy \( V_f \) since the firm does not have to pay the severance tax if the worker is not hired. On the other hand, if a firm is bargaining the wage with a continuing permanent contract employee (an insider), then the threat point is \( V_f - \Psi \) because the firm ends up with an unfilled vacancy and the obligation to pay the severance tax if the worker is dismissed. Finally, if an unemployed worker meets a firm with a temporary contract, the threat point is simply the value of an unfilled vacancy for this type of contract \( V_f \). The total surplus from a match for \( i = OP, IP, T \) \((S_i(x)) \) is defined as the sum of the values to the firm and the worker net of their values of continued search and payroll taxes. Therefore:

\[
S_{TP}(x) = (W_{OP}(x) - U)^\beta_P (I_0(x) - V_f)
\]

\[
S_{TP}(x) = (W_{OP}(x) - U)^{\gamma_P} (I_0(x) - V_f + \Psi)
\]

\[
S_{TP}(x) = (W_{OP}(x) - U)^{\gamma_P} (I_0(x) - V_f - \Psi)
\]

The solutions of the above optimization problems split the total surplus in fixed proportions at all points in time and at all \( x \geq x^* \) for \( i = OP, IP, T \). In each case, the proportions of the total surplus that goes to the workers are:

\[
W_{OP}(x) - U = \beta_P S_{TP}(x)
\]

\[
W_{IP}(x) - U = \beta_P S_{IP}(x)
\]

\[
W_{T}(x) - U = \beta_P S_{T}(x)
\]

while the firms obtain:

\[
I_0(x) - V_f = (1 - \beta_T^T) \frac{\lambda_T}{1 + \phi_T} S_{TP}(x)
\]

\[
I_0(x) - V_f + \Psi = (1 - \beta_T^T) \frac{\lambda_T}{1 + \phi_T} S_{IP}(x)
\]

\[
I_0(x) - V_f - \Psi = (1 - \beta_T^T) \frac{\lambda_T}{1 + \phi_T} S_{T}(x)
\]

\(^{17}\)Wages are bargained when an unemployed worker meets a firm (outsider permanent or temporary) and when a shock arrives (insider permanent).
Using Eqs. (13) and (14) to rewrite the flow values of workers and firms, Eqs. (1) to (7), in terms of the total surplus, and making the appropriate substitutions, it is easy to show that the wage equations are:

$$w_{wp}(x) = \frac{\beta(x - \lambda + \Psi) + (1 - \beta) \lambda + \phi}{1 - \phi} r U$$

(15)

$$w_{yp}(x) = \frac{\beta(x + r \Phi) + (1 - \beta) \lambda + \phi}{1 - \phi} r U$$

(16)

$$w_{y}(x) = \frac{\beta(x + (1 - \beta) \lambda + \phi)}{1 - \phi} r U - \lambda \int_{0}^{x} (W_{wp}(x') - U f_{wp}(x') dx')$$

(17)

Eqs. (15) and (16) are very similar to those found by Albrecht et al. (2009). The only difference is that, in this paper, workers also pay payroll taxes. In all cases, the wage is a weighted average of the match-specific productivity (adjusted by the severance tax in the case of permanent workers) and the worker’s outside option (the flow value of unemployment). Since $w_{wp}(x) = w_{yp}(x) = \frac{\beta(x - \lambda + \Psi) + (1 - \beta) \lambda + \phi}{1 - \phi} r U$ and that $(r, x, \beta, \Psi, \Phi)$ are all positive and $0 \leq \phi \leq 1$, the wage of a continuing employee (insider) with a permanent contract is higher than that earned by a new hire (outsider) with a similar type of contract (that is, $w_{yp}(x) > w_{wp}(x)$). This reflects the fact that a continuing employee has a better bargaining position with respect to the firm than a new hire because of the severance tax. Equation (17) is slightly different because in the case of temporary workers, the worker’s outside option has to take into account the possibility of reallocation to a permanent contract job. In particular, the outside option in this case is lower than the flow value of unemployment, in the expected capital gain, when there is a direct transition to a permanent job. It also reflects the fact that the worker is willing to accept a job with a lower wage because there is a change to enhance his position in the future with a permanent contract job.

Once again, using the workers’ and firms’ flow values written in terms of the total surpluses, it is straightforward to verify that:

$$S_{wp}(x) = \frac{1 - x_{wp}^* \lambda}{(1 - \eta) \lambda + \phi + (1 + \lambda \Psi) \Phi}$$

(18)

$$S_{yp}(x) = \frac{1 - x_{yp}^* \lambda}{(1 - \eta) \lambda + \phi + (1 + \lambda \Psi) \Phi}$$

$$S_{y}(x) = \frac{1 - x_{y}^* \lambda}{(1 - \eta) \lambda + \phi + (1 + \lambda \Psi) \Phi}$$

At this point, a discussion on the optimality of the match formation decision rule, which has a reservation value property, is necessary. So far it is assumed that this decision rule is optimal. It is evident from Eqs. (18) and (13), that both the total surplus function and the value of employment are strictly increasing in productivity $x$. Since the value of unemployment is constant, there is a reservation productivity $x_{y}^*$ such that $W(x) = U$, for $i = OP, IP, T$. Moreover, at that productivity the total surplus is zero ($S(x_{y}^*) = 0$). Using the flow values for an insider worker with a permanent contract, the wage equation, the total surplus definitions, and the condition $S(x_{y}^*) = 0$, it is possible to verify that:

$$x_{wp}^* = \frac{(1 + \phi)}{(1 - \eta)} r U - \frac{\lambda}{r + \lambda \Phi} \int_{x_{wp}}^{x_{y}^*} (x' - x_{wp}^* f_{wp}(x')) dx'$$

(19)

Define $T(x_{wp}^*)$ equal to the left hand side of Eq. (19). Note that $T: \mathbb{R} \rightarrow \mathbb{R}$ and that it is differentiable. The function $T(x_{wp}^*)$ is a contraction on $\mathbb{R}$ with respect to the usual metric if there is a real number $\epsilon \leq 1$ such that the derivative $|T'(x_{wp}^*)| < \epsilon$ for all $x_{wp}^* \in \mathbb{R}$. Note that $T'(x_{wp}^*) = \frac{\lambda}{r + \lambda \Phi} (1 - f_{wp}(x_{wp}^*)) < \epsilon \leq 1$ if $r + \lambda \Phi f_{wp}(x_{wp}^*) > 0$, which is true given the possible values of the model parameters. The direct application of the contraction mapping theorem implies that the equation $x_{wp}^* = T(x_{wp}^*)$ has a unique solution in $\mathbb{R}$.

In the same way, the flow value of an outsider permanent worker, the wage equation, the definition of total surplus, and the condition $S(x_{yp}^*) = 0$ can be used to find an expression for the reservation productivity of this type of worker. Additionally, Eq. (19) can assist in writing the resulting expression as:

$$x_{yp}^* = x_{y}^* + (r + \lambda) \Psi$$

(20)

Note that $x_{yp}^* \geq x_{wp}^*$, which once again reflects the better bargaining position of the insider worker. Also, since $x_{y}^*$ is uniquely determined, so is $x_{yp}^*$. Finally, the flow values of temporary contracts and the wage equation, together with the definition of the total surplus and the condition $S(x_{yp}^*) = 0$, generate:

$$x_{y}^* = \frac{(1 + \phi)}{(1 - \eta) \lambda + \phi + (1 + \lambda \Psi) \Phi} r U - \frac{\lambda}{r + \lambda \Phi} \int_{x_{y}^*}^{x_{y}^*} (x' - x_{y}^* f_{yp}(x')) dx'$$

(21)

The reservation productivity for a temporary contract is equal to the outside option of the worker (that is, the flow value of the unemployment state discounted by the expected capital gain in case of a direct transition to a permanent contract job), adjusted for payroll taxes. As before, since $x_{yp}^*$ is uniquely determined, so is $x_{y}^*$.

To close the model, the free-entry condition in the vacancy creation problem for both types of contracts is used. Profit maximization requires that all rents from new job creations should be exhausted such that the value of one more vacancy is zero, that is $V_{1} = 0$ for $i = OP, T$ (Mortensen and Pissarides, 1994). Applying this condition to Eqs. (8) and (9) and the definitions of total surplus in Eq. (18), the following equations can be obtained:

$$k_{op} = \frac{m \Psi q}{2 \Phi} \int_{x_{op}}^{x_{y}^*} (x - x_{op}^*) f_{op}(x) dx$$

(22)

$$k_{t} = \frac{m (1 - \eta) \Psi q}{2 \Phi} \int_{x_{t}}^{x_{op}} (x - x_{t}^*) f_{t}(x) dx$$

(23)

which implicitly defines a system of equations in $q$ and $\eta$. These last two expressions and the definition of the total surplus can be used to rewrite the flow value of unemployment in Eq. (1) in the following way:

$$r U = b + \left( \frac{1 - \tau_{e}}{1 + \phi r} \right) \frac{m \Psi q}{2 \Phi} \int_{x_{t}}^{x_{y}^*} (x - x_{t}^*) f_{t}(x) dx$$

(24)

A formal definition of the steady-state equilibrium can now be stated:

Definition 1. Given a vector of parameters $(b, \lambda_{p}, \lambda_{t}, \Psi, r, \beta_{p}, \beta_{t}, k_{op}, k_{t})$, a matching function $m(\cdot)$, a vector of taxes $(\tau_{p}, \tau_{t}, \Psi, \phi_{p}, \phi_{t}, \Phi, \Psi)$, and probability distribution functions for the productivity of permanent and temporary contracts $F_{p}(x)$ and $F_{t}(x)$, a steady-state equilibrium in a dual labor market economy is a labor market tightness $q$ and a proportion of job vacancies with permanent contracts $\eta$, together with reservation productivities $x_{i}^{*}$ for $i = OP, IP, T$, unemployment rate $u$ and employment rates $\phi_{p}$ and $\phi_{t}$ such that:

1. Given $q$ and $\eta$, and $rU$ from Eq. (24), the reservation productivities $x_{i}^{*}$ for $i = OP, IP, T$ solve Eqs. (19) to (21).
2. Given the reservation productivities $x_{i}^{*}$ for $i = OP, IP, T$, the unemployment rate $u$ and employment rates $\phi_{p}$ and $\phi_{t}$ satisfy Eqs. (10) to (12).

Note that the solution is unique given the value of $rU$, which is a function of the endogenous variables $q$ and $\eta$ (as mentioned in the next subsection).

The algorithm to computationally implement the model comes directly from the definition of steady-state equilibrium.
3. \( q \) and \( \eta \) solve the system of Eqs. (22) to (23) and are consistent with the reservation productivities \( s_i \) for \( i = OP, T \).

The equilibrium exists if the system of Eqs. (22) to (23) has a solution for \( q \) and \( \eta \) in the third part of Definition 1, which in turn depends entirely on the matching function (recall that \( s_i \) for \( i = OP, IP, T \) is given in this stage). Under the assumptions made for the matching function, particularly the one on its increasing characteristic, there is a solution possibly involving a corner solution in \( \eta \). If, in addition, it is assumed that the matching function is strictly increasing, then that solution is unique.

3. Data

To estimate the model, this paper uses microdata on the Chilean labor market, particularly, the longitudinal Social Protection Survey (Encuesta de Protección Social or EPS) from the Subsecretaria de Previsión Social (2002)\(^{22}\). This survey, which interviewed persons over the age of 18 years in 2002, 2004, 2006 and 2009, builds a panel of labor histories. In each survey, interviewers explicitly asked about the events (states in the labor market, monthly wages and weekly hours worked in each job) occurring in the years after the last survey in which the individual participated. A feature that makes this survey very attractive is its longitudinal dimension, one that is not commonly found in Latin American countries’ datasets. Even though the model to be estimated does not have on-the-job search, which makes the data on labor market histories in the employment state less relevant, the longitudinal dimension provides valuable information on transitions from the unemployment state to temporary and permanent contract jobs and from temporary to permanent contract jobs. This is important for the identification strategy used in the next section.

The estimation of the search model considers the survey of 2006\(^{23}\). The last spells of surveys 2004 and 2002 where used to reduce (or eliminate) the number of left censored spells. Since the model assumes ex-ante homogeneous workers, some observed heterogeneity controls are necessary to guarantee a certain degree of homogeneity consistent with the model assumptions. In particular, the estimation sample satisfies the following criteria: males between the ages of 25 and 55 years actively participating in the labor market and without a college degree. Initially, there were 16443 individuals in the 2006 survey and only 4487 had these characteristics. The literature that estimates a cross-section of workers in employment and unemployment states (Eckstein and van den Berg, 2007). Therefore, following this literature, a cross-section sample comprised of all labor market states (unemployment and employment states) was used.

The sample size was further reduced due to other problems with the data. First, double censored spells (or very long spells) in the unemployment state cannot be used because they generate an identification problem as discussed in the next section\(^{24}\). Fortunately, this type of spell represented only 0.4% of the valid sample, and could be discarded. Second, the sample contained unrealistically high wages. Therefore, to avoid this outlier problem, 2.5 of the upper and lower percentiles in wages were dropped from the sample (resulting in a reduction of 13% of the valid sample observations). This elimination

\(^{22}\) The survey is conducted by the Microdata Center of the Economics Department at the University of Chile with the participation of academics of the University of Pennsylvania and the University of Michigan.

\(^{23}\) The 2009 survey is contaminated with the recent recession, which started in 2008. The estimations use only unemployment spells with durations less than 50 months. Eliminating some unemployment information does not affect the sample representativity because the proportion of unemployed individuals remains close to that reported in the CASEN 2006.

\(^{24}\) The average unemployment duration is high when compared to that of the 2006 CASEN survey, in which the average unemployment duration is only 2.7 months. It is well known that one of the most important problems encountered when working with self-reported data is the quality of the information, where short lived events tend to be over reported. The problem is exacerbated when the self-reported data is retrospective as is the case in the EPS. However, given its longitudinal dimension, which is central for the identification of the parameters, this paper uses the EPS, even though the CASEN has a larger sample size and is more accurate (it is self-reported, but not retrospective).

\(^{25}\) CASEN 2006.

\(^{26}\) This average unemployment duration is high when compared to that of the 2006 CASEN survey, in which the average unemployment duration is only 2.7 months. It is well known that one of the most important problems encountered when working with self-reported data is the quality of the information, where short lived events tend to be over reported. The problem is exacerbated when the self-reported data is retrospective as is the case in the EPS. However, given its longitudinal dimension, which is central for the identification of the parameters, this paper uses the EPS, even though the CASEN has a larger sample size and is more accurate (it is self-reported, but not retrospective).

\(^{27}\) The low percentage of temporary contracts underestimates the importance of temporary workers when compared with that of the 2006 CASEN survey (Table 1).
(10% goes towards retirement, 7% towards health and approximately 3% towards disability) and are paid entirely by the worker (Edwards and Edwards, 2000). On the other hand, the unemployment insurance contribution depends on who pays the tax. In particular, workers hired under permanent contracts pay 0.6% of their wages to the unemployment insurance, while employers contribute 1.6% towards this insurance. If a worker is hired under a temporary contract, only the employer contributes 3% of the wages towards unemployment insurance (Fajnzylber et al., 2009).

Finally, the EPS survey contains information on the reasons for job termination and whether a severance payment occurred or not. Therefore, in principle it is possible to calculate the severance payment. However, since information on wages and duration are required and durations are likely to be (left-) censored, the average firing cost will be underestimated. In addition, there are other firing costs that are not considered in the data. Hence, in this paper the firing cost is expressed in terms of the average wage of permanent contract jobs, that is, $\Psi = \Pi \varphi$, where an estimate of $\Gamma$ is obtained from external sources. The World Bank (2005) Doing Business Project estimates a firing cost of 52 weeks for Chile ($\varphi \approx 12$ months)$^{28}$, which is in line with the ones used in the literature for Latin American countries; for example, Bosch and Esteban-Pretel (2012) use a proportion of 15 months of average wages of formal jobs in the case of Brazil.

4. Estimation

The model is estimated by maximum log-likelihood method using supply side information of the labor market, that is, durations in different labor market states and wages under temporary and permanent contracts. While this information, as pointed out by Flinn (2006), is useful in learning about arrival and termination rates, and the parameters that characterize the productivity distribution, it is not useful in characterizing the vacancy creation problem. Hence, the lack of demand side information is clearly a limitation. Since the market tightness ($\eta$) and the proportion of vacancies with permanent contracts ($\eta$) affect only the arrival rates $a_{wT}$ and $a_{T}\eta$, it is possible to estimate them as parameters, after which $\eta$ and $\eta$ can be recovered by relying on other sources of information or by making specific assumptions regarding the matching function. Consequently, the vacancy cost parameters can also be estimated. This is one of the alternative identification strategies proposed by Flinn (2006) to estimate search and matching models with endogeneous arrival rates only with supply side information. The identification of arrival and termination rates and productivity distribution parameters relies on Flinn and Heckman (1982), and since the model differentiates between insider and outsider permanent workers, a feature that is unobserved in the data, the estimation also relies on Flabbi (2010) strategy to identify a mixture of distributions.

4.1. The likelihood function

The data consists of unemployment durations, hourly wages and durations in jobs with temporary and permanent contracts, transitions from unemployment to both types of jobs, and transitions from temporary contract jobs to permanent contract jobs that is:

\[ \{t_{wT}, I(u \rightarrow eT), I(u \rightarrow eF)\}_{u \notin U}, \]

\[ \{w_{wT}, t_{wT}, I(eT \rightarrow eF), I(eT \rightarrow eF)\}_{e \notin \mathcal{E}}. \]

To find the unemployment duration contribution to the likelihood function, the hazard rate out of unemployment is defined as:

\[ h_u = a_{wT}^u[1 - F(x_{wp})] + a_{T}^u[1 - F(x_{wp})]. \tag{25} \]

In other words, the hazard rate is defined as the probability that a job is created once a worker meets an employer with any type of contract (reflected as a productivity drawn from the match greater than the reservation productivity). The hazard rate, conditional on the model, is constant. This implies that the contribution of the unemployment duration is the density of a negative exponential random variable with a coefficient equal to the hazard rate (Flabbi, 2010). Given that the unemployment duration is observed only for workers who are currently unemployed, the contribution of unemployment duration has to be weighted by the probability of being unemployed (the unemployment rate):

\[ f_u(t_{u, w} \mid t \in U) = f_u(t_{u, w} \mid t \in U) Pr(u \in U) = h_u \exp(-h_{u}t_{u}) \quad t_{u} > 0 \]

Using the idea of multiple-exit duration models of Bover and Gómez (2004), it is possible to distinguish between exits from unemployment to permanent and temporary jobs. Let the indicator variables of exit to permanent and temporary employments be denoted by $D_{p} = I(u \rightarrow eT)$ and $D_{T} = I(u \rightarrow eF)$, respectively. Then, it is possible to define the following hazard rates:

\[ h_u^P = a_{wT}^u[1 - F(x_{wp})] \]

\[ h_u^F = a_{T}^u[1 - F(x_{wp})] \]

such that the hazard rate out of unemployment is $h_u = h_u^P + h_u^F$. Therefore, the contribution of unemployment duration to the likelihood function becomes:

\[ f_u(t_{u, w} \mid t \in U) = \{h_u^P \exp(-h_u^P t_{u})\} D_{p} \{h_u^F \exp(-h_u^F t_{u})\} D_{T} \quad t_{u} > 0 \tag{26} \]

In turn, the hazard rate out of temporary contracts jobs, conditional on the model, is defined as $h_T = \lambda_T + \lambda_T\eta$, which can alternatively be written as

\[ h_T = \lambda_T + \lambda_T F(x_{wp}) + \lambda_T(1 - F(x_{wp})). \tag{27} \]

Considering again the idea of multiple-exit duration models, it is clear that the first two elements of Eq. (27) correspond to the hazard rate out of temporary jobs to unemployment, while the last term corresponds to the hazard rate out of these jobs to permanent contract jobs, that is:

\[ h_T^P = \lambda_T + \lambda_T F(x_{wp}) \]

\[ h_T^F = \lambda_T(1 - F(x_{wp})). \]

Since the hazard rate out of temporary jobs does not have duration dependence, by definition, it is possible to define the probability of transition to a permanent job in terms of the previous hazard rates:

\[ Pr(e_T \rightarrow e_F) = \frac{h_T^F}{h_T^P + h_T^F}. \tag{28} \]

Moreover, since the model does not explicitly incorporate on-the-job search in its structure, information on employment duration under temporary contracts is typically not used in the estimation (Eckstein and van den Berg, 2007). Therefore, in order to incorporate information about direct transitions in the estimation of $\lambda_T$ and $\lambda_T\eta$, Eq. (28) is used as a constraint in the maximization of the likelihood function, approximating $Pr(e_T \rightarrow e_F)$ using the data on transitions from temporary to permanent jobs.

There are three features of the data that need to be considered in order to derive the contribution of wages to the likelihood function. First of all, wages are observed in the data, but productivity is not. Secondly, observed wages are accepted wages. Finally, this information is available only for currently employed workers. Therefore, and following Flabbi (2010), the first step in finding the wages contribution...
is to map the unconditional wage cumulative distribution from the unconditional productivity cumulative distribution, and construct the truncated version of the density of the former distribution taking into account the optimal decisions of the agent in the model (that is, the wage equations and reservation productivities). The second step is to find the corresponding wage density and weight it by the probability of being employed (the employment rate). The wages contribution to the likelihood function, conditional on being a newly hired worker (outsider) with a permanent contract, is30:

\[
g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), OP) = \frac{(1 + \phi_F)^{\beta_F}}{\beta_F} \Phi \left( \frac{w_i - (1 - \beta_F)(1 + \phi_F)U + \phi_U}{\beta_F} \right)_{OP} - \frac{(1 - \beta_F)(1 + \phi_F)}{(1 - \tau_F)}_{OP} \epsilon_{OP} \tag{29}
\]

On the other hand, the wages contribution to the likelihood function, conditional on being a continuing employee (insider) with a permanent contract, is:

\[
g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), IP) = \frac{(1 + \phi_F)^{\beta_F}}{\beta_F} \Phi \left( \frac{w_i - (1 - \beta_F)(1 + \phi_F)U - \phi_U}{\beta_F} \right)_{IP} - \frac{(1 - \beta_F)(1 + \phi_F)}{(1 - \tau_F)}_{IP} \epsilon_{IP} \tag{30}
\]

Eqs. (29) and (30) are conditioned on observing wages for new hires and for continuing employees, both under permanent contracts. However, information about the identity of the type of worker is not available in the data. Therefore, one additional step in the construction of the likelihood contribution of wages is necessary for this type of contract. To remove the condition of whether the worker with a permanent contract is an outsider or an insider (considering that \(w_p(x^*_{ip}) = w_p(x^*_{ip})\)), the following expression is used:

\[
g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), P) = g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), OP) \text{Pr}(OP) + g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), IP) \text{Pr}(IP) \tag{31}
\]

The probability of being a new hire (outsider) is \(\text{Pr}(OP)\), and it depends on the duration of the job since the worker remains an outsider if there are no productivity shocks, but the longer the contract lasts the more likely it is for productivity shocks to arrive. Productivity shocks, conditional on the model, are governed by a Poisson process. Therefore, \(\text{Pr}(OP) = \text{Pr}(\text{receive 0 shocks in } t) = \exp(-\lambda_F t)\). Also note that \(\text{Pr}(IP) = 1 - \text{Pr}(OP)\). Using these probabilities, the last equation becomes:

\[
g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), P, t_{ip}) = \exp(-\lambda_F t_{ip}) \frac{(1 + \phi_F)^{\beta_F}}{\beta_F} \Phi \left( \frac{w_i - (1 - \beta_F)(1 + \phi_F)U + \phi_U}{\beta_F} \right)_{OP} - \frac{(1 - \beta_F)(1 + \phi_F)}{(1 - \tau_F)}_{OP} \epsilon_{OP} + \exp(-\lambda_F t_{ip}) \frac{(1 + \phi_F)^{\beta_F}}{\beta_F} \Phi \left( \frac{w_i - (1 - \beta_F)(1 + \phi_F)U - \phi_U}{\beta_F} \right)_{IP} - \frac{(1 - \beta_F)(1 + \phi_F)}{(1 - \tau_F)}_{IP} \epsilon_{IP} \tag{31}
\]

which is a mixture of two truncated distributions with a weight equal to the probability of being an outsider worker. The construction of the likelihood contribution of wages, conditional on being a temporary worker, follows the procedure described above and can be written as:

\[
g(w_i, i \in E_i | w_i > w_p(x^*_{ip}), T) = \frac{(1 + \phi_F)^{\beta_F}}{\beta_F} \Phi \left( \frac{w_i - (1 - \beta_F)(1 + \phi_F)U + \phi_U}{\beta_F} \right)_{OP} - \frac{(1 - \beta_F)(1 + \phi_F)}{(1 - \tau_F)}_{OP} \epsilon_{OP} + \frac{(1 + \phi_F)^{\beta_F}}{\beta_F} \Phi \left( \frac{w_i - (1 - \beta_F)(1 + \phi_F)U - \phi_U}{\beta_F} \right)_{IP} - \frac{(1 - \beta_F)(1 + \phi_F)}{(1 - \tau_F)}_{IP} \epsilon_{IP} \tag{32}
\]

where

\[
r_U = U - \frac{\lambda_F \phi_U}{r - \phi_U} \int_{0}^{\infty} (x - x^*_{ip}) \phi_U(x') dx' \tag{33}
\]

Using the densities in Eqs. (26), (31), and (32), the likelihood function is:

\[
L(\Theta; w, t) = \prod_{i=1}^{N} \left[ \mathbb{I}_{i}(t_i, w_i, i \in U) \right] \mathbb{I}_{i}(t_i, w_i, i \in U) \int_{x_{ip}}^{\infty} \left( x - x^*_{ip} \right) \phi_U(x') dx' \tag{33}
\]

where \(\Theta\) is the vector of parameters, \(t_{ip}\), \(w_{ip}\), \(x_{ip}\) are unemployment duration, wages and employment duration under permanent contracts, respectively, \(u_i=1\) if unemployed and 0 otherwise, and \(e_{ip}=1\) if the individual is employed with a permanent contract and 0 otherwise. Note that the employment duration of a job with a permanent contract indirectly contributes to the likelihood function, through the wage contributions, and that employment duration under temporary contracts does not provide useful information to the likelihood.

The reservation productivities are endogenous variables in the model and in order to choose the vector of parameters \(\Theta\), the likelihood in Eq. (33) has to be maximized, also subject to the following equilibrium conditions:

\[
x_{ip}^* = \frac{(1 + \phi_F)U - r_U + r + \phi_F}{r - \phi_U} \int_{x_{ip}}^{\infty} \left( x - x^*_{ip} \right) \phi_U(x') dx' \tag{33}
\]

Finally, \(r_U\) is also an endogenous variable in the model, but for estimation purposes it is treated as a constant31. Therefore, \(b\), which is the only parameter that does not appear directly in the likelihood, is chosen so that all equilibrium conditions are met as described below.

4.2. Identification

The identification strategy has three stages. The first follows Flinn and Heckman (1982) and Flabbi (2010) and is related to the identification of the parameters in the likelihood function (Eq. (33)), which are the job arrival rates \((a_{ij}, a'_{ij})\), the productivity and termination shock arrival rates \((\lambda_{ij}, \lambda'_{ij})\), the reservation productivities \((x^*_{ip}, x^*_{ip}', x^*_{ip}'')\), the flow value of unemployment \((rU)\), and the parameters governing the productivity distributions \((F_{ij}(x), F_{ij}(x))\).32

Following Flinn and Heckman (1982), a necessary condition for the identification of the parameters in the likelihood function is the recoverability condition of the productivity distribution. Under this condition, the entire wage distribution, and thereby the productivity distribution, should be uniquely recoverable from a truncated distribution with a known truncation point. In addition, according to Flabbi (2010) the necessary condition to identify a mixture of two truncated distributions, such as the likelihood contribution of permanent workers, discussed in the previous subsection, is that the productivity distribution belongs to a location-scale family. In this paper, it is assumed that the match-specific productivity in both types of contracts

\[30\] The detailed derivations of the likelihood contributions are presented in Appendix A.

\[31\] This is a common practice in the literature. See, for example, Eckstein and van den Berg (2007).

\[32\] The formal proof of identification for the first stage of the strategy is presented in Appendix B.
and Using statistical significance, any additional parameters in the matching function and \( \eta \), \( \beta \), and \( q \) can be solved in the following way. First, proposed by Flinn (2006), consists of using a matching function that does not contain any unknown parameters. A good option, which fulfills the assumptions made in Section 2, is the exponential matching function \( m(u, v) = (1 - e^{-uv})^{-\gamma} \). The second consists of using external sources to obtain estimates of a Cobb-Douglas matching function parameter.35 For the case of the Chilean labor market, the elasticity of the matching function was estimated using the procedure proposed by Borowczyk-Martins et al. (2013), and time series on vacancies, unemployment and arrival rate of jobs.37 The resulting matching function is: \( m(u, v) = \beta \cdot \delta \cdot \mu \). In any case, once the matching function is identified, all demand side parameters are identified.

Identification and (consistent) estimation of the parameters \( q \), \( \eta \), \( \beta \), \( k_p \) and \( k_T \) and build on the consistent estimators of the parameters \( \alpha^P \), \( \alpha^T \), \( \lambda_p \), \( \lambda_T \), \( \lambda_r \), \( \lambda_x \), \( \lambda^p \), \( \lambda^T \), \( \lambda^r \) in the following way. First, \( \eta \) and \( q \) solve \( \lambda^P = m(\eta \mu) \) and \( \lambda^T = m(1 - \eta \mu) \) provided that the matching function \( m(\cdot) \) is identified. Second \( k_p \) and \( k_T \) solve:

\[
\begin{align*}
    k_p &= \frac{m(\eta \mu)(1 - \eta \mu)}{\eta \mu} \int_0^\infty (x - \eta \mu) f_p(x) dx \\
    k_T &= \frac{m(1 - \eta \mu)(1 - \eta \mu)}{\eta \mu} \int_0^\infty (x - \eta \mu) f_T(x) dx
\end{align*}
\]

Finally, once all the above parameters are identified, \( b \) can be recovered using the equilibrium condition:

\[
\begin{align*}
    \frac{\partial m}{\partial \mu} &= \frac{\partial m}{\partial \eta} = \frac{\partial m}{\partial b} = 0
\end{align*}
\]
b = r\tilde{U} - \left(1 - \tau_p\right) \frac{\hat{\Theta}_{\beta} \hat{b}_p}{(1 - \beta_p)} - \left(1 - \tau_{p}\right) \frac{1}{1 + \varphi_{p}} \left(1 - \hat{\Theta}_{\beta} \hat{b}_p\right) \\

4.3. Econometric issues

Three econometric issues arise in estimating the model: (1) measurement error in wages, (2) censoring in unemployment duration data, and (3) censoring in employment duration data. This subsection explains how each of these issues are dealt with.

Measurement error in wages is incorporated in the estimation procedure for two reasons. First, it is very likely that wages are measured with error since the wage data is self reported and it includes wages from past years. Second, and most important, it is not possible to estimate the reservation productivities using the lowest observed wage in both types of contracts, in the spirit of Flinn and Heckman (1982), because the mapping between the reservation productivity and the reservation wage, in the case of permanent jobs, depends on other parameters to be estimated (the relations are implied in the equilibrium conditions). This problem is critical because the reservation productivities are the truncation parameters in the accepted wage distributions. Therefore, changing these parameters in the maximization process of the likelihood function changes its support, which violates one of the regularity conditions. A way to avoid this problem is to introduce measurement error. Following Flinn (2002) and Flabbi and Leonardi (2010), it is assumed that the measurement error is multiplicative, and therefore the observed wage \( w^o \) can be expressed as:

\[ w^o = w \cdot \varepsilon \]

where the measurement error \( \varepsilon \) is log-normally distributed:

\[ \varepsilon = \phi \left( \frac{\ln(\varepsilon) - \mu}{\sigma} \right) \theta_{\sigma} \]

In order to restrict the number of parameters to estimate, it is assumed that the conditional expectation of the observed wages is equal to the true wages, as is done in Flinn (2002); that is, \( E[w^o|w] = w \), which implies that \( E[\varepsilon|w] = 1 \). This assumption together with the log-normal assumption implies that the parameters \( \mu \) and \( \sigma \) satisfy \( \sigma = \sqrt{2} \mu \). Therefore, only one parameter of the measurement error distribution has to be estimated.

Given the wage density functions for jobs with permanent and temporary contracts, \( g(w, i \in E_P | \lambda) = \gamma(w, \lambda) \), and \( g(w, i \in E_T | \lambda) = \gamma(w, \lambda) \), respectively, and the error density function \( m(\varepsilon) \), the implied density functions of observed wages can be written as:

\[ g^o_P(w^o) = \frac{1}{\gamma(w, \lambda)} \int g(w, i \in E_P | \lambda) \cdot \gamma(w, \lambda) \cdot \theta_{\sigma} \cdot \exp(-\frac{\ln(\varepsilon) - \mu}{\sigma}) \cdot \sigma \cdot \theta_{\sigma} \]

\[ g^o_T(w^o) = \frac{1}{\gamma(w, \lambda)} \int g(w, i \in E_T | \lambda) \cdot \gamma(w, \lambda) \cdot \theta_{\sigma} \cdot \exp(-\frac{\ln(\varepsilon) - \mu}{\sigma}) \cdot \sigma \cdot \theta_{\sigma} \]

Censoring in unemployment duration data is potentially very problematic because it can generate identification problems and bias the estimated parameters. In particular, if the unemployment spells are double censored, that is right and left censored at the same time, the identification of the parameters in the likelihood estimation is not possible because permanent unemployment can be generated by a different combination of parameters (Flinn, 2002), hence the reason for not using double censored spells in the estimation. The estimated parameters will be biased when there are right or left censored spells. Fortunately, controlling for these two types of censoring is straightforward when the spells are exponentially distributed, and can easily be incorporated in the likelihood function. Because there are no left censored unemployment spells in the sample, this article only describes right censoring correction. Let \( c_i \) be indicator variables taking the value of 1 if the unemployment spell is right censored and zero otherwise. The likelihood contribution of a complete unemployment spell is:

\[ f_d(t_{i, u}, i \in U, c_i = 0) = h_u \exp(-h_{t_{i, u}}) \cdot t_{i, u} \]

while the likelihood contribution of a right censored unemployment spell is:

\[ f_d(t_{i, u}, i \in U, c_i = 1) = \Pr(T > t_{i, u}) = \exp(-h_{t_{i, u}}) \cdot t_{i, u} \]

Taking into account measurement error in wages and censoring in unemployment spells, the likelihood function becomes:

\[ L(\Theta;\; w,\; t) = \prod_{i=1}^{N} \left[ \int f_d(t_{i, u}, i \in U, c_i = 0) \right]^s \cdot \int f_d(t_{i, u}, i \in U, c_i = 1)^{1-s} \cdot \Pr(T > t_{i, u}) \cdot P[w_{i}] \cdot d(w_{i}) \]

which is maximized to choose \( \Theta \); subject to equilibrium constraints:

\[ x^*_{dP} = \frac{1 + \lambda_{dP}}{1 - \lambda_{dP}} U - r - \lambda_{dP} \int_{t_{dP}}^{\infty} (x' - x^o_{dP}) f_p(x') dx' \]

\[ x^*_{dP} = x^*_{dP} + (\lambda_{dP} + r) \Psi \]

\[ x_{dP} = \frac{1 + \lambda_{dP}}{1 - \lambda_{dP}} U - r - \lambda_{dP} \int_{t_{dP}}^{\infty} (x' - x^o_{dP}) f_p(x') dx' \]

and

\[ \Pr(\varepsilon_t = \varepsilon_0) = \frac{\lambda_{\varepsilon} (1 - F_{\varepsilon}(\varepsilon))}{\lambda_{\varepsilon} + \lambda_{\varepsilon}} \]

The last econometric issue is related to the censoring problem in the employment duration data. In this paper, only employment spells of jobs with permanent contracts are relevant. Recall that employment duration indirectly contributes to the likelihood function through the wage contribution, because it affects the probability of being an outsider (that is, \( \Pr(OP) = \Pr(t_{i, P} \text{ receive 0 shocks in } t) = \exp(-\lambda_{t_{i, P}}) \)). As previously mentioned, employment duration spells can be left or right censored. Right censored spells do not represent a problem because the probability of receiving a determined number of shocks before time \( t \) is what is important in the model; hence, at that time the future is irrelevant. On the other hand, left censored spells do represent a potential problem. This can be observed by expressing the \( \Pr(t_{i, P} \text{ receive 0 shocks in } t) \) such that the distinction is made between the observed duration \( t_{i, P}^o \) and the true duration \( t_{i, P}^t \). Since \( t_{i, P}^o = t_{i, P} - a \), where \( a \geq 0 \), then:

\[ \Pr(OP) = \exp(-\lambda_{t_{i, P}^o} + a) = \exp(-\lambda_{t_{i, P}^t} + a) \exp(-\lambda_{t_{i, P}}) \]

Given that \( \exp(-\lambda_{t_{i, P}}) \leq 1 \) with \( \lambda_{t_{i, P}} \geq 0 \) and \( a \geq 0 \), it is clear that if \( a \) is not taken into account, then the probability of receiving 0 shocks in \( t \) is overestimated. In the case of permanent contracts, this affects the weights in the mixture of wage densities, which in turn can potentially lead to a bias problem in the estimation.

The censoring problem in the employment duration data is neglected in the estimation results presented in the next subsection since the probability of receiving zero shocks in \( t \) exponentially decreases with employment duration. Hence, the effect of the additional months in the duration of long spells is not important. This is the case in the data used in the estimations, since the left censored spells duration is at least 17 months (and there are less than 2% of these spells).

4.4. Estimation results

Table 2 reports the estimation results. The first two rows show the
Table 2
Estimated Parameters.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Std.Err. (*i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{w*}$</td>
<td>0.0728</td>
</tr>
<tr>
<td>$a_{u*}$</td>
<td>0.1132</td>
</tr>
<tr>
<td>$\lambda_{p}$</td>
<td>0.0028</td>
</tr>
<tr>
<td>$\lambda_{T}$</td>
<td>0.0078</td>
</tr>
<tr>
<td>$b_{n}$</td>
<td>0.0039</td>
</tr>
<tr>
<td>$\gamma_{p}$</td>
<td>1.0425</td>
</tr>
<tr>
<td>$\gamma_{T}$</td>
<td>1.5198</td>
</tr>
<tr>
<td>$\sigma_{w}$</td>
<td>1.2496</td>
</tr>
<tr>
<td>$\sigma_{u}$</td>
<td>1.9576</td>
</tr>
<tr>
<td>$\sigma_{b}$</td>
<td>0.2480</td>
</tr>
<tr>
<td>$\mu_{p}$</td>
<td>-0.5372</td>
</tr>
<tr>
<td>$\mu_{T}$</td>
<td>1.5312</td>
</tr>
<tr>
<td>$\sigma_{x}$</td>
<td>0.8385</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>3600</td>
</tr>
<tr>
<td>Loglik</td>
<td>-7972</td>
</tr>
<tr>
<td>F-test $a_{w*}^*$</td>
<td>1.09</td>
</tr>
<tr>
<td>F-test $a_{u*}^*$</td>
<td>100.32</td>
</tr>
<tr>
<td>$\sigma_{w}^*$</td>
<td>0.3694</td>
</tr>
<tr>
<td>$\sigma_{u}^*$</td>
<td>0.1133</td>
</tr>
<tr>
<td>$\lambda_{p}$</td>
<td>85.1735</td>
</tr>
<tr>
<td>$k_{p}$</td>
<td>27.1933</td>
</tr>
<tr>
<td>$b$</td>
<td>-3.4474</td>
</tr>
</tbody>
</table>

Note: $P(\gamma \to \gamma)$ was set to 0.0942 as in the data.

Table 3
Technological and Preference Parameters.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Std.Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobb Douglas Matching Function (*)</td>
<td>0.3694</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3694</td>
</tr>
<tr>
<td>$q$</td>
<td>0.1133</td>
</tr>
<tr>
<td>$k_{p}$</td>
<td>85.1735</td>
</tr>
<tr>
<td>$k_{p}$</td>
<td>27.1933</td>
</tr>
<tr>
<td>$b$</td>
<td>-3.4474</td>
</tr>
<tr>
<td>Exponential Matching Function</td>
<td>0.3913</td>
</tr>
<tr>
<td>$q$</td>
<td>0.1860</td>
</tr>
<tr>
<td>$k_{p}$</td>
<td>17.1535</td>
</tr>
<tr>
<td>$b$</td>
<td>-3.4474</td>
</tr>
</tbody>
</table>

Note: Standard Errors calculated using delta method.

The productivity shock arrival rate for permanent jobs, reported in Table 2, indicates that productivity shocks do not occur very often. Similarly, the termination rate for the temporary jobs, reported in Table 2, shows a high persistence. Clearly, direct transition from temporary contract jobs to unemployment occurs twice as fast as the opportunities to be reallotted to a permanent job. As a result, the hazard rates out of employment, shown in Table 2, are low. When comparing these hazard rates with their data counterparts, as shown in Table 4, it is clear that the shock arrival rate for permanent jobs and the termination rate for temporary jobs are underestimated, as are the correspondent hazard rates out of employment. However, it is important to note that the hazard rates observed in the data are not directly comparable with those of the model because, in the absence of on-the-job search, the interpretation of the employment duration in the model is career duration instead of job duration. Another explanation of these results is directly related with the data. As mentioned in Section 3, the data on unemployment and employment durations has a retrospective and self-reported nature and seems to be over reported compared to another.

Table 3 shows the estimated values of the technological parameters (the proportion of permanent vacancies, the market tightness and the flow cost of vacancies in temporary and permanent jobs) and the preference parameter (the flow value of leisure) using both the Cobb-Douglas and the Exponential matching functions. All the results discussed below are conditional on the particular assumptions made about the matching function. First, the proportion of permanent job vacancies in the market is between 37 to 39% depending on the matching function used. Second, the estimated market tightness differs between matching functions. In particular, the market tightness, along with the unemployment rate in Table 4, imply that the total vacancy rate of the economy ($\gamma + \gamma_{T}$) is 0.5% when the Cobb-Douglas function is used, and 1% when the Exponential function is used. Third, under the Cobb-Douglas matching function, the flow costs of permanent and temporary jobs are around 83 and 27 U.S. dollars, respectively. Meanwhile, these same flow costs are around 47 and 17 U.S. dollars under the Exponential matching function. In any case, it is clear that maintaining a permanent job vacancy unfilled is substantially (between 2.7 and 3.1 times) more expensive than maintaining a temporary job vacancy unfilled. Finally, the flow disutility of leisure is around 3.4 U.S. dollars per hour and it does not depend on which matching function is used to identify it.

 footnotes: 40 Comparing this result with that found by Flinn (2002), who estimates a flow vacancy cost of 128 U.S. dollars for the U.S. economy for 1996, suggests that the cost of an unfilled vacancy of a permanent job is substantially lower in the Chilean labor market. However, the difference is not that clear relative to the average wage (while in the U.S. economy it is 18 times the average wage, in Chile it is between 18 and 30 times the average wage of a permanent worker depending on the matching function used).
Table 4
Predicted Values.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Err. (%)</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(y_{e})$</td>
<td>1.456</td>
<td>0.1272</td>
<td>n.a.</td>
</tr>
<tr>
<td>$V(y_{e})$</td>
<td>11.039</td>
<td>1.6561</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E(y_{p})$</td>
<td>1.380</td>
<td>0.2928</td>
<td>n.a.</td>
</tr>
<tr>
<td>$V(y_{p})$</td>
<td>1.944</td>
<td>0.3258</td>
<td>n.a.</td>
</tr>
<tr>
<td>Offered Wages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(u_{0,pe})$</td>
<td>1.413</td>
<td>0.0699</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E(u_{0,pe})$</td>
<td>1.538</td>
<td>0.0711</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E(u_{1,pe})$</td>
<td>1.196</td>
<td>0.1384</td>
<td>n.a.</td>
</tr>
<tr>
<td>Accepted Wages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(u_{0,pe})$</td>
<td>2.696</td>
<td>0.0536</td>
<td>2.554</td>
</tr>
<tr>
<td>$E(u_{1,pe})$</td>
<td>2.578</td>
<td>0.0472</td>
<td>2.554</td>
</tr>
<tr>
<td>$E(u_{1,pe})$</td>
<td>1.652</td>
<td>0.0313</td>
<td>1.604</td>
</tr>
<tr>
<td>Labor Market Status</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.052</td>
<td>0.0036</td>
<td>0.055</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.712</td>
<td>0.0075</td>
<td>0.706</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.236</td>
<td>0.0072</td>
<td>0.239</td>
</tr>
<tr>
<td>Labor Market Dynamics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{0}$</td>
<td>0.073</td>
<td>0.0057</td>
<td>0.075</td>
</tr>
<tr>
<td>$b_{1}$</td>
<td>0.002</td>
<td>0.00002</td>
<td>0.012</td>
</tr>
<tr>
<td>$b_{2}$</td>
<td>0.012</td>
<td>0.00130</td>
<td>0.041</td>
</tr>
</tbody>
</table>

* Standard Errors calculated using delta method.

To conclude this section, some specification tests that were performed and an assessment of the fit of the model are discussed. The last two rows of the bottom panel of Table 2 report the statistics of two F-tests. The first test corresponds to the null hypothesis that both types of jobs have the same arrival rate, which implies that the proportion of vacancies is 50% for each type of job. This null hypothesis cannot be rejected. Using the asymptotic standard errors of the arrival rates in Table 2, the hypothesis that the arrival rates of temporary and permanent jobs are equal to zero can be tested. The results indicate that both hypotheses are also rejected at 1% significance level. The second test tries to verify whether the productivity in each type of job is drawn from the same distribution (given that in both cases log-normality is imposed). Once again, the data does not support the hypothesis (at 1% significance level) that the productivity in both types of jobs come from the same distribution.

Table 4 reports the predictions of the model and some comparable moments in the data. On one hand, in terms of fit, the average wages predicted by the model are very close to their sample counterparts. In turn, Fig. 4 shows that the good fit on wages is not only observed at the mean but also at the distribution level. On the other hand, model predictions of the unemployment and employment rates are really close to those observed in the data. The hazard rate out of unemployment also fits the data well. However, the model predictions of hazard rates out of employment do not fit the data well, as was previously discussed.

5. Counterfactual and policy experiments

The counterfactual experiment consists of comparing the benchmark economy, that is, the one characterized by the estimated parameters and in which temporary contracts are allowed, with an economy in which the use of temporary contracts is not allowed by law. In the latter economy, the model is solved assuming $\gamma = 1$ and using all other estimated parameters (except those related with temporary jobs). The policy experiment consists of analyzing the impact of changes in the firing cost on the two economies previously mentioned by taking into account the equilibrium effects. In particular, the experiment analyzes the effect of changes in the firing cost within a range of zero to twice the benchmark severance tax. In conducting the counterfactual and policy experiments, a Cobb-Douglas matching function is used to solve the model with the estimated elasticity of $\gamma = 0.8255$. In both exercises, it is possible to analyze the effect, under alternative institutional arrangements of more stringent labor protection, on labor market dynamics (that is market tightness, availability of vacancies of both types of jobs, arrival rates, hazard rates out of unemployment and employment, and unemployment and employment rates) and on productivity and wages (reservation productivities, average offered and accepted wages, and inequality between workers with different types of contracts).

5.1. Labor market dynamics

The first row in Table 5 shows that at any firing cost, the labor market is tighter when temporary contracts are not allowed, that is there are fewer jobs available relative to the number of searchers. In particular, when these contracts are not allowed, the market tightness is at least 45% lower than when they are allowed. This is explained by the fact that the presence of temporary contracts increases the vacancies available in the market. Analyzing the effect of an increase in the firing cost, the same table shows that the market tightness decreases with the firing cost when temporary contract jobs are not allowed because this cost makes vacancy creations of permanent jobs less attractive. Quantitatively, the effect is not substantial -going from no firing cost to twice the benchmark firing cost leads to a decrease of 1.8% in the market tightness. On the contrary, when temporary contracts are allowed, the effect of the firing cost on the market tightness is the opposite. Indeed, despite the fact that the firing cost makes permanent job vacancy creation less attractive, they make temporary job vacancy creation more attractive and in the end this latter effect dominates the former one. This is observed in the decreasing proportion of permanent job vacancies shown in the second row of Table 5. Market tightness increases by more than 6.4%, going from no firing cost to twice the benchmark firing cost and the proportion of permanent vacancies decreases by almost 8 percentage points. Permanent vacancies can disappear if the firing costs are really high (more than 10 times that of the benchmark). This is possible in the model but not plausible in practice.

The arrival rates for permanent and temporary jobs, shown in the

---

41 Simulated wages are constructed using the productivity distributions for both types of contracts and wages equations. In the case of permanent contract workers, simulated wages are constructed as a weighted average of simulated wages of outsider and insider workers, where the weight is given by the probability of being an outsider worker.

42 Note that the market tightness is defined as the vacancy to unemployment ratio, and therefore a higher ratio indicates that the labor market is less tight.
Table 5
Counterfactual and Policy Experiments.

<table>
<thead>
<tr>
<th></th>
<th>TC Allowed</th>
<th></th>
<th>TC Not Allowed</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 × Y</td>
<td>1 × Y</td>
<td>2 × Y</td>
<td>0 × Y</td>
</tr>
<tr>
<td>Market Tightness</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>0.1096</td>
<td>0.1133</td>
<td>0.1167</td>
<td>0.0691</td>
</tr>
<tr>
<td>η</td>
<td>0.4101</td>
<td>0.3694</td>
<td>0.3341</td>
<td>1</td>
</tr>
<tr>
<td>Arrival Rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a_{u,p}^T)</td>
<td>0.0772</td>
<td>0.0728</td>
<td>0.0687</td>
<td>0.1101</td>
</tr>
<tr>
<td>(a_{u,p}^T)</td>
<td>0.1043</td>
<td>0.1132</td>
<td>0.1214</td>
<td>n.a.</td>
</tr>
<tr>
<td>Labor Market Status</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>0.0543</td>
<td>0.0522</td>
<td>0.0492</td>
<td>0.0487</td>
</tr>
<tr>
<td>(σ_p)</td>
<td>0.7248</td>
<td>0.7119</td>
<td>0.7082</td>
<td>0.9513</td>
</tr>
<tr>
<td>(σ_T)</td>
<td>0.2209</td>
<td>0.2358</td>
<td>0.2425</td>
<td>n.a.</td>
</tr>
<tr>
<td>Hazard Rates</td>
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<td></td>
</tr>
<tr>
<td>h_{1T}</td>
<td>0.0700</td>
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<td>0.0369</td>
</tr>
<tr>
<td>h_{0T}</td>
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<td>0.0017</td>
<td>0.0019</td>
</tr>
<tr>
<td>h_0</td>
<td>0.0117</td>
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<td>0.0117</td>
<td>n.a.</td>
</tr>
<tr>
<td>Reservation Productivity</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_{1T}^p)</td>
<td>1.2354</td>
<td>1.2696</td>
<td>1.3038</td>
<td>1.0383</td>
</tr>
<tr>
<td>(s_{1T}^p)</td>
<td>1.2354</td>
<td>1.0198</td>
<td>0.8042</td>
<td>1.0383</td>
</tr>
<tr>
<td>(s_{2T}^p)</td>
<td>1.0643</td>
<td>1.0425</td>
<td>1.0242</td>
<td>n.a.</td>
</tr>
<tr>
<td>Accepted Wages</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E(w_{0</td>
<td>p,T}))</td>
<td>2.7214</td>
<td>2.6962</td>
<td>2.6729</td>
</tr>
<tr>
<td>(E(w_{0</td>
<td>p,T}))</td>
<td>2.7214</td>
<td>2.5785</td>
<td>2.4325</td>
</tr>
<tr>
<td>(E(w_{0</td>
<td>p,T}))</td>
<td>1.6765</td>
<td>1.6526</td>
<td>1.6325</td>
</tr>
<tr>
<td>Inequality</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outsider/Insider</td>
<td>1.000</td>
<td>1.046</td>
<td>1.099</td>
<td>1.000</td>
</tr>
<tr>
<td>Temporary/Insider</td>
<td>0.616</td>
<td>0.613</td>
<td>0.611</td>
<td>n.a.</td>
</tr>
<tr>
<td>Temporary/Insider</td>
<td>0.616</td>
<td>0.641</td>
<td>0.671</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

The third and fourth rows of Table 5, reflect what was discussed above. When temporary jobs are allowed, as the firing cost increases, fewer vacancies for permanent jobs reduce the rate at which they arrive, while more temporary job vacancies accelerate their arrival rate. The reduction in the case of permanent job vacancies is 14.3% and the increase in the case of temporary job vacancies is 16.3%. When temporary jobs are not allowed, the arrival rate of permanent jobs also falls with the firing cost but the size of the effect is much smaller—it only reduces permanent job vacancies by 1.3%.

Rows five to seven in Table 5 show the effect of firing costs on the different labor market states, that is, on the unemployment and the employment rates, under both types of contracts and their corresponding durations. The fifth row shows that the unemployment rate falls with more stringent employment protection when temporary contracts are not allowed. In this case the unemployment rate falls by 1.2 percentage points. This indicates that the effect of the firing cost in reducing the job destruction rate dominates that which discourages employment creation. The hazard rate out of unemployment, shown in the eight row of Table 5, does not change when temporary contracts are not allowed suggesting that there is no effect on unemployment duration. When temporary contracts are allowed, the effect of stringent protection on the unemployment rate is attenuated (unemployment rate falls by only 0.5 percentage points), indicating that the effect of the flows out of temporary jobs dominates that of the flows out of unemployment into temporary contracts. Additionally, the hazard rate out of unemployment increases, suggesting that an additional positive impact of temporary jobs is that they help to reduce unemployment duration by a month when employment protection becomes more stringent. A note on what is behind these results is necessary. The difference between the reservation productivities between outsiders and insiders increases more that 0.5 dollars per hour (regardless of the existence of temporary jobs) and represents the effect of (discourage) job destruction. In turn, the reservation productivity of outsiders permanent only increases by less than 1 cent per hour when temporary contracts are not allowed and in 7 cents per hour when they are allowed, representing in this case the effect of (discourage) job creation. The former effect reflects the fact that the firm is willing to keep a worker even if he is less productive to avoid the firing cost, while the latter effect occurs because permanent jobs become more scarce (employers become more picky when hiring and there is substitution of vacancies between types of jobs). Note that the existence of temporary contract jobs makes permanent (and more productive) jobs relatively more scarce and increases their productivity requirement and the discouragement of job creation in permanent contract jobs. This explains why temporary jobs mitigate the effect of firing costs on unemployment.

The sixth row of Table 5 shows that the employment rate in permanent jobs increases by 1.2 percentage points, which is consistent with the decrease in the unemployment rate when temporary contracts are not allowed. However, the fact that the employment rate of permanent jobs falls by more than 2 percentage points when temporary contracts are allowed, implying a substitution between permanent and temporary jobs, is more interesting (recall that the unemployment rate falls slightly in this scenario). The ninth row of Table 5 shows the hazard rate out of permanent jobs. As expected, more stringent protection in permanent jobs discourages its destruction and this is true regardless of whether the use of temporary contracts is allowed or not. Quantitatively, the duration of permanent jobs increases by 17.6 and 35.7% when temporary contracts are allowed and when they are not, respectively. Finally, the seventh row of Table 5 shows that the employment rate in temporary jobs increases in almost 2.4 percentage points when the firing cost rises. This is consistent with the substitution effect previously mentioned. This positive relationship between temporary job shares and employment protection corresponds to the model counterpart of Fig. 1. Furthermore, row ten in Table 5 shows the hazard rate out of temporary jobs, which is constant by construction (the termination rate in the model, comprised by the termination and reallocation shocks, is exogenous).

5.2. Productivity and wages

Row eleven in Table 5 shows how the reservation productivity of new hires with permanent contracts (outsiders) changes with the firing cost. Regardless of the existence of temporary contracts, the firing cost discourages new hires by increasing the threshold at which matches are formed. When temporary contracts are not allowed, this reservation productivity increases by less than 0.7% with the firing cost, while when both types of contracts coexist it increases by 5.5%. The existence of temporary contracts exacerbates the negative effect on job creations. Row twelve in Table 5 shows the reservation productivity of continuing employees with permanent contracts (insiders). In this case, the reservation productivity decreases with the firing cost and the effect is quantitatively important (it falls by more than 40%). This is explained by the fact that more protection generates a higher bargaining advantage for workers, reducing the firms’ outside option. Therefore, firms are willing to maintain a larger proportion of their workers even if they become less productive after a shock has occurred. For both, insiders and outsiders, the reservation productivity is always higher when temporary contracts are allowed, indicating that a higher productivity is sustained with permanent jobs when the two types of contracts coexist. Row thirteen in Table 5 shows the reservation productivity of temporary jobs. The positive effect on unemployment of increasing the firing cost in permanent jobs is that the lower reservation productivity facilitates job creation. The down side is that lower productivity jobs will be created.

Rows fourteen to sixteen in Table 5 show the average accepted wages. In the case of permanent jobs, the firing cost affects average accepted wages through three mechanisms. First, they directly affect the total surplus of the match. Second, they have an equilibrium effect on the flow value of unemployment. Third, they have an equilibrium effect on the conditional average productivity through the reservation values. In the case of temporary jobs, average accepted wages are affected by the equilibrium effects on the flow value of unemployment.
and the reservation productivity. In that Table it is shown that the equilibrium effects of the firing cost on reservation productivities are important. Indeed, they almost offset the direct effect of the firing cost for new hires (the decrease is only between 1.7 and 5.1%) and substantially reduce the average accepted wages for continuing employees (between 10.6 and 15.6%). Row sixteen shows a fall of 2.6% in average accepted wages for temporary jobs. Hence, the effect on the reservation productivity also contributes to this fall. Finally, accepted wages are higher when temporary jobs are allowed. This is due to a higher reservation productivity, in the case of average accepted wages.

This subsection concludes with the effect of the firing cost on inequality. In this paper, inequality is defined as the difference in wage rates of permanent and temporary workers. Therefore, in this exercise both types of contracts are allowed. Rows eighteen and nineteen in Table 5 show the ratios between the average wage in temporary jobs and the average wage in permanent jobs for new hires and for continuing employees. There are three comments worth mentioning from the analysis of that ratio. First, inequality is high since the gap between accepted wages for permanent and temporary workers is around 40% (comparing new hires in temporary jobs and continuing employees in permanent jobs, respectively). Second, the pattern of the wage ratios is consistent with the changes in wages given by changes in the firing costs. Finally, although inequality changes with the firing cost, it remains high for the range of firing costs considered in this paper, suggesting that the effect of this policy is limited in this aspect.

5.3. Welfare analysis

Following Flinn (2006) and Flabbi (2010), this paper exploits the stationary nature of the model to analyze the long-run welfare impact of changes in the policy parameters (mainly the firing cost) under the two different assumptions regarding the labor market institution: when temporary contracts are allowed and when they are not. To define a long-run measure of welfare, it is important to recall that at any point in time workers are unemployed, employed under a permanent contract or employed under a temporary contract. Similarly, at any point in time firms can have permanent or temporary job vacancies filled or they can be searching to fill their vacancies. The latter is not taken into account because unfilled vacancies have, by definition, a value of zero (free-entry condition). In this context, the following Social Welfare function is defined:

\[ S(t, \phi, \Psi^t) = \int_{t_1}^{t_2} \left[ \frac{j(t)}{T(t)} \int_{t_1}^{t_2} \left( \frac{j(t)}{T(t)} \right) \left( 1 - F(t) \right) \right] dt \]

where: \( t = (t_1, t_2) \), \( \phi = (\phi_1, \phi_2) \), \( U(t, \phi, \Psi^t) \) is the unemployed agents' welfare, \( \Psi^t(t, \phi, \Psi^t) \) is the average workers' welfare (\( j = OP, IP, T \) ) and \( T(t, \phi, \Psi^t) \) is the average welfare of firms with filled vacancies (\( j = OP, IP, T \)). Note also that \( e_{op}(t, \phi, \Psi^t) = e_{op}(t, \phi, \Psi^t)Pr(1-Pr(1-Pr)) \) and \( e_{ip}(t, \phi, \Psi^t) = e_{ip}(t, \phi, \Psi^t)(1-Pr(1-Pr)) \). To implement Eq. (34) it is necessary to define the individual contribution to the Social Welfare function:

\[ U(t, \phi, \Psi^t) = \int_{t_1}^{t_2} \left[ \frac{j(t)}{T(t)} \int_{t_1}^{t_2} \left( \frac{j(t)}{T(t)} \right) \left( 1 - F(t) \right) \right] dt \]

\[ W(t, \phi, \Psi^t) = \int_{t_1}^{t_2} W_t(t) \left[ \frac{j(t)}{T(t)} \right] \int_{t_1}^{t_2} \left( \frac{j(t)}{T(t)} \right) \left( 1 - F(t) \right) dt \]

\[ J(t, \phi, \Psi^t) = \int_{t_1}^{t_2} J_t(t) \left[ \frac{j(t)}{T(t)} \right] \int_{t_1}^{t_2} \left( \frac{j(t)}{T(t)} \right) \left( 1 - F(t) \right) dt \]

Eq. (34) is then used to evaluate changes in welfare (total, workers' and firms' welfares) when the firing cost changes in the case where temporary contracts are allowed and in the case where they are not allowed. Note that Eq. (34) is the analog to the criterion used by Hosios (1990) in his labor market efficiency study when two types of jobs exist.

Fig. 5 shows the level of welfare reached when temporary contracts are not allowed and when they are allowed, for each degree of labor protection. Note that, for any firing cost below half of the benchmark firing cost, the total welfare is greater in an economy without temporary contracts. The second observation that can be made is that the relative welfare decreases when the firing cost increases. When the firing cost is very low, the level of welfare is higher in an economy without temporary contracts. In this case what matters is the possibility of productivity gains in permanent contracts. However, when firing costs are rather high, the level of welfare in an economy with both types of contracts increases (reducing the relative welfare) and the degree of flexibility becomes more valuable. Also, recall that temporary jobs can be reallocated to permanent jobs upon termination, although this does not occur very often. Temporary contracts make agents better off if the firing cost reaches high levels (including the benchmark). Finally, the shape of the relative welfare means that stringent labor protection generates important trade-offs in terms of productivity gains and flexibility.

It is worth mentioning that the assumption behind welfare calculations is that the collected taxes, both payroll and severance, are not redistributed among workers and are just thrown into the ocean. In essence, they are pure taxes. Removing this assumption is not trivial for the bargaining problem but a discussion regarding how the results would change is important. In particular, when the firm pays the firing taxes to the worker leaving a permanent contract job, the value of that job should increase because that value will take into account, in addition to income flows and continuations values, the present value of the firing taxes. Moreover, as the firing taxes increase, the importance of the present value of those future payments should increase more than proportionally with respect to the flow income (given that in this model the firing taxes are just constant). On the side of the firm nothing changes because they pay the total cost of firing a worker regardless of the destination of that payment. Therefore, in equilibrium, a higher value of the permanent job should make the hiring process (reducing the reservation values for new permanent jobs) easy and speed up the transition from unemployment to permanent jobs.

One of the results found in the previous section is that in equilibrium the permanent contract jobs increase their relative availability in the market as the firing cost increases, reducing the arrival rate of permanent contract jobs and making the match formation of this type
of job difficult. Note that when it is assumed that firing taxes are paid to the worker, these two effects are the opposite and therefore the role of temporary jobs in slowing down the hiring process in the permanent contract jobs should be attenuated. In terms of Fig. 5, the temporary contracts would probably generate welfare gains as the firing cost becomes more stringent but these gains should not increase without bound and instead they should disappear for really high values of the firing cost.

6. Concluding remarks

This paper presents a search and matching model with the following features: First, it has a dual labor market represented by two types of contracts, permanent and temporary, and the availability of both is endogenously determined as part of the equilibrium. Second, labor protection is incorporated in the form of firing costs to analyze its relationship with the equilibrium share of temporary contracts. Finally, it incorporates the possibility of productivity gains in permanent jobs and reallocation shocks in temporary jobs to account for possible transitions to permanent jobs. This model is structurally estimated using likelihood methods for the Chilean labor market. In the estimation procedure only supply side data is used and the identification strategy, particularly for the technological or demand side parameters, is discussed. Finally, counterfactual and policy experiments are performed to quantitatively evaluate the role of labor protection legislation and the relative availability of temporary contracts in unemployment, welfare, and inequality.

The estimation results indicate that around 40% of the available vacancies are for permanent contracts. This reflects large differences in vacancy costs (3 times higher for permanent vacancies). In terms of the dynamics of the labor market, the magnitude of the parameters suggests that temporary jobs arrive more frequently than permanent jobs (although this difference does not seem statistically relevant) and that workers meeting both types of vacancies draw, on average, similar productivities. With respect to wages, workers receive wage offers that are, on average, 18% higher when they meet firms with permanent contracts than when they meet firms with temporary ones. Once the job is accepted that difference becomes 63%, on average. Finally, the long run unemployment rate is about 5.2%.

The counterfactual and policy experiments results indicate that when the firing cost increases, fewer permanent job vacancies reduce the rate at which they arrive, while more temporary job vacancies accelerate its arrival rate. Temporary jobs magnify the effect of firing costs on permanent job arrival rates. Even though labor protection is useful in reducing unemployment, temporary contracts balance out this effect leaving unemployment almost unchanged. Meanwhile, labor protection increases the (equilibrium) employment rate in jobs with temporary contracts. The effects on employment and unemployment rates discussed above implies that there is a substitution effect between both types of jobs. With respect to inequality, the negative effect of firing costs on wages is barely compensated with the existence of temporary contracts. Hence, inequality is persistent. Finally, welfare analysis indicates that temporary contracts generate welfare gains as labor protection increases.

Some policy implications can be drawn from these results. First, temporary contracts increase flexibility and make agents better off when firing costs are high. It is important to remember that this is a steady-state result, which could be magnified if the cushion effect during business cycles is taken into account. Second, limiting the use of temporary contracts (in an extreme case, eliminating them) can increase welfare only if labor protection is not stringent. Therefore, stringent labor protection generates important trade-offs between productivity and flexibility. Hence, labor protection levels matter in terms of welfare.

Finally, although temporary contracts are also widely used in European countries, the results found cannot be directly extrapolated to the European case, not only because of the differences in the structure of the temporary contracts but also because the parameters governing the dynamics of the labor market differ. Many results depend heavily on the magnitude of parameters\(^{43}\).

### Appendix A. Wages contribution to the likelihood function

To find the wages distribution conditional on the model, the first step is to map the productivity distribution for each type of contract into an unconditional wages distribution. First, mapping the productivity distribution to a wage distribution for a new hire with permanent contract (outsider) gives:

$$G(w|OP) = \Pr(W \leq w|OP) = \frac{\Pr\left(\frac{\beta_p(x - \lambda_p W) + (1 - \beta_p)(1 + \phi_p)(1 - \tau_p)U}{1 + \phi_p} \leq w|OP\right)}{1 - \Pr\left(\frac{\beta_p(x - \lambda_p W) + (1 - \beta_p)(1 + \phi_p)(1 - \tau_p)U}{1 + \phi_p} \leq w|OP\right)}$$

$$= \frac{\Pr\left(\frac{w}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)(1 - \tau_p)U + \lambda_p W}{\beta_p} \leq 1\right)}{1 - \Pr\left(\frac{w}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)(1 - \tau_p)U + \lambda_p W}{\beta_p} \leq 1\right)}$$

Second, the wages distribution for continuing employees with permanent contracts (insiders) that result from the same mapping is:

---

\(^{43}\)Nevertheless, the modelling strategy can be useful in the European case because the definition of temporary contracts includes that of fixed-term contracts and, therefore, the model can be estimated using European data trying to match a hazard rate consistent with a 12 month duration contract.
\[ G(w|P, IP) = \Pr(W \leq w|IP) \]

\[
= \Pr\left( \beta_p (x + r\Psi) + (1 - \beta_p) \frac{(1 + \phi_p)}{(1 - \tau_p)} rU \leq w|IP \right)
\]

\[
= \Pr\left( x \leq \frac{(1 + \phi_p)}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)}{\beta_p} \frac{(1 - \tau_p)}{rU - r\Psi} \right)
\]

\[
= F\left( \frac{(1 + \phi_p)}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)}{\beta_p} \frac{(1 - \tau_p)}{rU - r\Psi} \right)
\]

Finally, in the case of a temporary job, the mapping gives:

\[ G(w|T) = \Pr(W \leq w|T) \]

\[
= \Pr\left( \beta_T x + (1 - \beta_T) \frac{(1 + \phi_T)}{(1 - \tau_T)} \frac{rU}{1 - \tau_T} \leq w|T \right)
\]

\[
= \Pr\left( x \leq \frac{(1 + \phi_T)}{\beta_T} w - \frac{(1 - \beta_T)(1 + \phi_T)}{\beta_T} \frac{(1 - \tau_T)}{rU} \right)
\]

\[
= F\left( \frac{(1 + \phi_T)}{\beta_T} w - \frac{(1 - \beta_T)(1 + \phi_T)}{\beta_T} \frac{(1 - \tau_T)}{rU} \right)
\]

where

\[
rU = rU - \frac{\lambda_p}{\beta_p} \frac{(1 - \tau_p)}{r + \lambda_p (1 + \phi_p)} \int_{\tau_T}^{\infty} (\phi - \tau_T) f_p(\phi') \phi' \, d\phi'
\]

Note that in the data the distributions of accepted wages are observed. These distributions, conditional on the model, are truncations of the above unconditional wages distributions and the truncation point is the reservation wage (this value is also a mapping from the reservation productivity using wages equations). Conditioning on wages above the reservation value and taking into account that wages are observed only for those who are employed, the following is obtained:

\[
g(w|h) > w_{IP}(x_{IP}^*) \text{, OP, } i \in E_P = \frac{(1 + \phi_p)}{\beta_p} f_p\left( \frac{w}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)}{\beta_p} \frac{(1 - \tau_p)}{rU + \lambda_p \Psi} \right)
\]

\[
g(w|h) > w_p(x_p^*) \text{, IP, } i \in E_P = \frac{(1 + \phi_p)}{\beta_p} f_p\left( \frac{w}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)}{\beta_p} \frac{(1 - \tau_p)}{rU} \right)
\]

\[
g(w|h) > w_T(x_T^*) \text{, } T, i \in E_T = \frac{(1 + \phi_T)}{\beta_T} f_T\left( \frac{w}{\beta_T} - \frac{(1 - \beta_T)(1 + \phi_T)}{\beta_T} \frac{(1 - \tau_T)}{rU} \right)
\]

Removing the condition of being an employee and using the probability of having a permanent or a temporary contract (that is, the equilibrium employment rate in each type of contract) result in:

\[
g(w, i, E_P|h) > w_{IP}(x_{IP}^*) \text{, OP} = \frac{(1 + \phi_p)}{\beta_p} f_p\left( \frac{w}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)}{\beta_p} \frac{(1 - \tau_p)}{rU + \lambda_p \Psi} \right)
\]

\[
e_{IP}g(w, i, E_P|h) > w_p(x_p^*) \text{, IP} = \frac{(1 + \phi_p)}{\beta_p} f_p\left( \frac{w}{\beta_p} - \frac{(1 - \beta_p)(1 + \phi_p)}{\beta_p} \frac{(1 - \tau_p)}{rU} \right)
\]

\[
e_Tg(w, i, E_T|h) > w_T(x_T^*) \text{, } T = \frac{(1 + \phi_T)}{\beta_T} f_T\left( \frac{w}{\beta_T} - \frac{(1 - \beta_T)(1 + \phi_T)}{\beta_T} \frac{(1 - \tau_T)}{rU} \right)
\]

The next step is to remove the condition of whether the worker with a permanent contract is an outsider or an insider. Using the fact that \( w_{IP}(x_{IP}^*) = w_p(x_p^*) = w_T(x_T^*) \), the density of permanent job’s wages is:

\[
g(w, i, E_P|h) > w_{IP}(x_{IP}^*) \text{, } P = g(w, i, E_P|h) > w_p(x_p^*) \text{, } P, OP)Pr(OP) + g(w, i, E_P|h) > w_p(x_p^*) \text{, } P, IP)Pr(IP)
\]

where \( Pr(OP) \) is the probability of receiving zero shocks in \( t \). This probability depends on the duration of the job. The more the contract lasts the less likely it is that no productivity shocks have arrived. Conditional on the model, productivity shocks arrive at a Poisson rate \( \lambda_p \), and therefore \( Pr(OP) = Pr[ \text{ receive 0 shocks in } t] = \exp(-\lambda_p t) \) and \( Pr(IP) = 1 - Pr(OP) \). Hence:
$(w_i, i \in E_\tau h_U > w_T(x_T^\tau), P)$

$$\begin{aligned}
\phi_{\text{eq}, x_T^\tau} &= \frac{\exp(-\lambda_T h_U)}{\beta_T} f_T \left( \frac{w_i + \phi_T}{\beta_T} - \frac{(1 - \beta_T) (1 + \phi_T) r T U + \lambda_T w_T}{\beta_T - (1 - T_U)} \right) \\
\alpha_{\text{eq}, x_T^\tau} &= \frac{\exp(-\lambda_T h_U)}{\beta_T} f_T \left( \frac{w_i + \phi_T}{\beta_T} - \frac{(1 - \beta_T) (1 + \phi_T) r T U - r T P}{\beta_T - (1 - T_U)} \right)
\end{aligned}$$

Appendix B. Parameter identification in the likelihood function

Identification is formally shown by closely following Flabbi (2010). Recalling that the likelihood function was:

$$L(\Theta^T; w, t) = \prod_{i=1}^{N} \left[ f_T(t, w_i, t \in U) \right] \times [g(w_i, i \in E_\tau h_U > w_T(x_T^\tau), P)]^{\rho(x^{1-\nu})} \times [g(w_i, i \in E_T h_U > w_T(x_T^\tau), T)]^{[1-\rho(x^{1-\nu})]}$$

or alternatively using logarithm:

$$\ln L(\Theta^T; w, t) = \sum_{i \in N_p} \ln[h_{i, p}^T \exp(-h_{i, p}^T t_{a, i})] + \sum_{i \in N_T} \ln[h_{i, T}^T \exp(-h_{i, T}^T t_{a, i})]$$

Using the contribution of unemployment duration and wages, the likelihood becomes:

$$\begin{aligned}
\ln L(\Theta^T; w, t) &= \sum_{i \in N_p} \ln[h_{i, p}^T \exp(-h_{i, p}^T t_{a, i})] + \sum_{i \in N_T} \ln[h_{i, T}^T \exp(-h_{i, T}^T t_{a, i})] \\
&+ \sum_{i \in N_p} \ln\sum_{i \in N_T} \ln\left\{ \frac{\exp(-\lambda_T h_U)}{\beta_T} f_T \left( \frac{w_i + \phi_T}{\beta_T} - \frac{(1 - \beta_T) (1 + \phi_T) r T U + \lambda_T w_T}{\beta_T - (1 - T_U)} \right) \right\} \\
&+ \sum_{i \in N_T} \ln\sum_{i \in N_p} \ln\left\{ \frac{\exp(-\lambda_T h_U)}{\beta_T} f_T \left( \frac{w_i + \phi_T}{\beta_T} - \frac{(1 - \beta_T) (1 + \phi_T) r T U - r T P}{\beta_T - (1 - T_U)} \right) \right\} \\
&+ \sum_{i \in N_p} \ln h_{p, T}^* + \sum_{i \in N_T} \ln h_{T, p}^*
\end{aligned}$$

where

$$\tilde{r} U = r U - \frac{\lambda_T h_U}{\beta_T} \left( 1 - T_U \right) \int_{x_T^\tau}^{\infty} (x' - x_T^\tau) f_T(x') dx'$$

Considering first the contribution of unemployment duration, the unemployment rate, and the employment rates in jobs with both types of contracts in Eq. (B.1):

$$N_p h_{p, T}^* - h_{p, T}^* \sum_{i \in N_p} t_{a, i} + N_T h_{T, p}^* - h_{T, p}^* \sum_{i \in N_T} t_{a, i} + N_p \ln h_{p, T}^* + N_T \ln h_{T, p}^* + N \ln h_{U, T}$$

The steady-state equilibrium conditions, Eqs. (10) to (12) of the paper, are:

$$\begin{aligned}
u &= \frac{h_{p, U} h_{T, T}}{h_{p, T} h_{U, T} + h_{p, U} h_{T, T} + h_{p, U} h_{T, U}} \\
\epsilon_p &= \frac{h_{p, T} h_{U, T} + h_{p, U} h_{T, U} + h_{p, U} h_{T, T}}{h_{p, T} h_{U, T} + h_{p, U} h_{T, U} + h_{p, U} h_{T, T}} \\
\epsilon_T &= \frac{h_{p, U} h_{T, T} + h_{p, U} h_{T, U} + h_{p, U} h_{T, T}}{h_{p, U} h_{T, T} + h_{p, U} h_{T, U} + h_{p, U} h_{T, T}}
\end{aligned}$$

where: $h_{p, U} = \alpha_p^\tau [1 - F_T(x_T^\tau)]$, $h_{T, U} = \lambda_T f_T(x_T^\tau)$, $h_{p, U} = \lambda_T f_T(x_T^\tau)$, $h_{U, T} = h_{T, U}$, and $h_T = \lambda_T + \lambda_P$. Replacing these equations, the following is obtained:

$$N_p h_{p, T}^* - h_{p, T}^* \sum_{i \in N_p} t_{a, i} + N_T h_{T, p}^* - h_{T, p}^* \sum_{i \in N_T} t_{a, i} + (N_p + N_T) \ln h_{p, T}^* + N_p \ln h_{U, T} + N_T \ln h_{T, p}^* + N_T \ln h_{U, T} + h_{p, U} h_{T, T} + h_{p, U} h_{T, T}$$
Now, taking the first order conditions with respect to the hazard rates:

\[
\begin{align*}
 h_{\text{UP}} & : \ N_{\text{u,p}} - \sum_{i \in N_{\text{u,p}}} t_{i,j} + \frac{N_{\text{u,p}}}{h_{\text{UP}} + h_{\text{TPU}}} - \frac{N_{h_{\text{TPU}}}}{h_{\text{UP}} + h_{\text{TPU}}} = 0 \\
 h_{\text{UT}} & : \ N_{\text{u,R}} - \sum_{i \in N_{\text{u,R}}} t_{i,j} + \frac{N_{\text{u,R}}}{h_{\text{UT}} + h_{\text{TPU}}} + \frac{N_{h_{\text{UT}}}}{h_{\text{UT}} + h_{\text{TPU}}} = 0 \\
 h_{\text{TP}} : \ N_{\text{h,TP}} - h_{\text{UP}}h_{\text{UT}}h_{\text{TPU}} = 0 \\
 h_{\text{PU}} : \ \frac{N_{\text{h,PU}}}{h_{\text{UT}}} + \frac{N_{h_{\text{UT}}}}{h_{\text{UT}}} = 0 \\
 h_{T} : \ \frac{N_{\text{h,T}}}{h_{T}} + \frac{N_{h_{\text{UT}}}}{h_{\text{UT}}} = 0
\end{align*}
\]

The system can be solved for the five unknowns. So the hazard rates out of unemployment and out of employment are identified just with unemployment duration data and the transitions from unemployment to both types of contracts. In terms of the model parameters:

\[
\begin{align*}
 h_{\text{UP}} = a_{\text{u}}^{*}[1 - F_{\text{u}}(x_{\text{u,p}})] \\
 h_{\text{TP}} = \lambda_{\text{u}}[1 - F_{\text{u}}(x_{\text{u,p}})] \\
 h_{\text{PU}} = \lambda_{\text{p}}F_{\text{u}}(x_{\text{u,p}}) \\
 h_{\text{UT}} = a_{\text{T}}^{*}[1 - F_{\text{T}}(x_{\text{T}})] \\
 h_{T} = \lambda_{T} + \lambda_{R}
\end{align*}
\]

To take into account the observed proportion of temporary job to permanent job transitions the following restriction is used:

\[
Pr(e_{T} \rightarrow e_{R}) = \frac{\lambda_{T}(1 - F_{\text{T}}(x_{\text{T}}))}{h_{T}}
\]

On the other hand, recalling from Eq. (B.1), the contribution to the likelihood of wage of temporary workers was:

\[
\sum_{i \in N_{T}} \ln \left[ \frac{(1 + \phi_{i})}{\beta_{T}} \frac{(1 + \phi_{i})(1 - \beta_{i})}{\beta_{T}} \frac{1}{1 - F_{T}(x_{i})} \right]
\]

Now, using location and scale parameters notation:

\[
\sum_{i \in N_{T}} \ln \left[ \frac{1}{\gamma_{T}} \frac{w_{i} - L_{T}}{S_{T}} \right]
\]

where:

\[
L_{T} = \frac{(1 - \beta_{i})}{(1 - \gamma_{T})} + \frac{\beta_{T}}{(1 + \phi_{i})}x_{i}^{*}
\]

\[
S_{T} = \frac{\beta_{T}}{(1 + \phi_{i})}a_{i}^{*}
\]

and:

\[
\overline{r}U = \frac{\lambda_{T} \beta_{T} (1 - \tau_{R})}{\tau_{R}} + \lambda_{T} (1 + \phi_{i}) \int_{x_{\text{u,p}}}^{\infty} (x' - x_{\text{u,p}}) f_{p}(x') dx'
\]

Note that \(L_{T}\) and \(S_{T}\) are identified from temporary jobs wage data. Finally, the wage contribution to the likelihood of workers with permanent contracts in Eq. (B.1) was:

\[
\sum_{i \in N_{p}} \ln \left[ \frac{\exp(- \lambda_{p}t_{i,p})}{\beta_{p}} \frac{(1 + \phi_{i})}{\beta_{p}} \frac{(1 - \beta_{i})}{\beta_{p}} \frac{(1 + \phi_{i})(1 + \psi_{i})}{(1 - \gamma_{p})U + \lambda_{p}\psi} \frac{1}{1 - F_{p}(x_{i})} \right] \\
+ \frac{1 - \exp(- \lambda_{p}t_{i,p})}{\beta_{p}} \frac{(1 + \phi_{i})}{\beta_{p}} \frac{(1 - \beta_{i})}{\beta_{p}} \frac{(1 + \phi_{i})(1 + \psi_{i})}{(1 - \gamma_{p})U - \lambda_{p}\psi} \frac{1}{1 - F_{p}(x_{i})}
\]
Now, using the location and scale parameters notation again, the following is obtained:

\[
\sum_{i \in \mathcal{L}^P} \ln \left( \frac{\exp(-\lambda^P x_i^P) \frac{1}{S_{\mathcal{L}^P}(x_i^P)} w_i - L_{\mathcal{L}^P}}{1 - F_{\mathcal{L}^P}(x_i^P)} \right) + (1 - \exp(-\lambda^P x_i^P) \frac{1}{S_{\mathcal{L}^P}} w_i - L_{\mathcal{L}^P}) \frac{1}{1 - F_{\mathcal{L}^P}(x_i^P)}
\]

where:

\[
L_{\mathcal{L}^P} = \left( 1 - \beta^P \right) \mu^P + \frac{\beta^P}{1 + \phi^P} r^P
\]

\[
L_{\mathcal{I}^P} = \left( 1 - \beta^P \right) rU - \mu^P + \frac{\beta^P}{1 + \phi^P} r^P
\]

\[
S_{\mathcal{L}^P} = \frac{\beta^P}{1 + \phi^P} \sigma^P
\]

The contribution of permanent job wages is a mixture of two truncated normal distributions that share the same scale parameter. Since the weights change in a deterministic way, Teicher (1963) result applies so \( L_{\mathcal{L}^P}, L_{\mathcal{I}^P}, S_{\mathcal{L}^P}, \lambda^P \) are identified from wage data. Finally, the model restrictions are:

\[
\begin{align*}
\alpha^P & = \left( 1 + \lambda^P \right) U - \mu^P + \frac{\lambda^P}{r + \lambda^P} \int_{\phi^P}^{\infty} (x' - \alpha^P \mu^P) f^P(x') dx'
\end{align*}
\]

\[
\begin{align*}
\alpha^P & = \chi^P + (\lambda^P + \gamma) \mu^P
\end{align*}
\]

\[
\begin{align*}
\alpha^P & = \left( 1 + \lambda^P \right) U - \mu^P + \frac{\lambda^P}{r + \lambda^P} \int_{\phi^P}^{\infty} (x' - \alpha^P \mu^P) f^P(x') dx'
\end{align*}
\]

It is possible to recover all the model parameters in the likelihood by solving Eqs. (B.2) to (B.14) for thirteen unknowns, \((\alpha^P, \alpha^P, \lambda^P, \lambda^P, \gamma, \mu^P, \chi^P, \chi^P, \phi^P, \sigma^P, \sigma^P, \sigma^P)\).

References


